Is Model-Theoretic Logical Consequence Analytic or Synthetic and How Can We Obtain Logical Necessity Through Analyticity?

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ABSTRACT

This paper reexamines the question of whether logic is analytic or synthetic under the model-theoretic conception of logical consequence, arguing that the issue has transformed rather than vanished. I argue that model-theoretic logical consequence, when interpreted through interpretational semantics, can be regarded as analytic in the sense that the meanings of the logical constants provide the semantic resources required for explaining logical necessity. Revisiting Etchemendy's distinction between interpretational and representational semantics, I argue that it mirrors the traditional analytic/synthetic divide: the former aligns with logical truths grounded in the meanings of logical constants, while the latter corresponds to truths dependent also upon the formal structure of the world. Etchemendy's claim that interpretational semantics cannot secure logical necessity therefore constitutes a challenge to the traditional thesis that logical necessity can be explained by analyticity. I also argue that Etchemendy's criticism ultimately reveals two semantic conditions that any adequate explanation of logical necessity must satisfy, and show how Gila Sher's influential formal-structural account secures these conditions by grounding logic in world's formal structure, thereby explaining logical necessity through its syntheticity. The paper concludes by suggesting that these same conditions can instead be grounded in the intensional meanings of the logical constants, thereby pointing toward a more promising analytic foundation for logical necessity.

1. Introduction

Nowadays not many people discuss whether logic is analytic or synthetic, in this paper I show that it is not that the answer to the question has been settled and logic is indubitably analytic (or synthetic), rather, the framework of the question

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itself has transformed in light of the modern contentions regarding the model-theoretic conception of logical consequence. Ultimately, this transformed framework provides a novel perspective on the classical question of logic's analytic or synthetic nature. More specifically, I argue that Etchemendy's (1990) distinction between interpretational semantics and representational semantics (On the former, models reinterpret the non-logical vocabulary against the actual world, whereas on the latter they represent alternative ways the world itself could be.) can be understood as a reformulation of the traditional analytic/synthetic distinction. On this understanding, interpretational semantics corresponds to the thesis that logical truths[1] are grounded in the meaning, whereas representational semantics corresponds to the view that logical truths also depend essentially on the structure of the world. Accordingly, Etchemendy's objection—that interpretational semantics cannot secure logical necessity—challenges the traditional view that logical necessity can be grounded in the analyticity of logic.

Throughout this paper, I understand the analyticity of logic in its traditional metaphysical sense: logical truths are true solely in virtue of the meanings (of the logical constants). Although other conceptions of analyticity—notably epistemic conceptions[2]—have been defended in literature, the present discussion concerns the metaphysical notion because it is the one most directly implicated by model-theoretic semantics.

There was a period of time when logic was treated as paradigmatically analytic in the analytic tradition, influenced by philosophers like Frege, Russell and early Carnap. But since Quine's challenge against the distinction between analytic and synthetic, doubts have been cast upon the view that logic is analytic, at least in the sense that logical truths can be true solely in virtue of meaning and independent of fact. For, according to Quine, 'analytic' depends on notions like synonymy, which are hard to define rigorously (Quine, 1951); and even logic could be revised when faced with recalcitrant experience, as classic logic might be replaced by Quantum logic if we are to make sense of quantum mechanics, resulting in a seamless web of belief. (ibid.)

Along with the development of modern logic, discussions regarding whether logic is analytic or not has been pushed to an increasingly peripheral

[1] Throughout this paper I will mostly focus on the case of logical truths for ease of exposition.

[2] One formulation of the epistemic conception holds that merely grasping a sentence's meaning is sufficient to justify believing it is true. For an argument advocating a shift in focus from metaphysical to epistemic analyticity, see Boghossian (1996).

position, but they certainly have not died out. In today's philosophical discussions about logic, we can find traces of this distinction in prevalent debates about logical anti-exceptionalism, logical realism and logical pluralism etc.[3] Driven by Tarski's seminal definitions of truth and logical consequence, the field of logic has increasingly shifted toward formal systems and model theory. Consequently, meaning analysis has been replaced by investigations that emphasize mathematical properties, such as 'truth in all models'. Nothing is felt to be lost in this practice, because Tarski's account is innately semantic. Indeed, his definition makes essential reference to satisfaction, a notion that fundamentally concerns the relationship between our language and the world.

However, objections have been raised regarding the adequacy of Tarski's account. Most influentially, Etchemendy (1990) introduces the distinction between interpretational semantics and representational semantics. Since Etchemendy interprets Tarski's semantics as fundamentally interpretational, he argues that this account leads to what he calls "Tarski's fallacy": Tarski commits a modal fallacy by shifting the necessity operator to the wrong position. More specifically, instead of establishing that Tarskian consequence implies necessary truth-preservation, Tarski's argument establishes only that, necessarily, Tarskian consequence implies material truth-preservation, thereby leaving the modal force of logical consequence unexplained. Closely connected with this diagnosis is what Etchemendy calls the reduction principle, according to which Tarski's account reduces logical truth (which is valid and necessary) to merely obtained state of affairs. The criticism is that truth in all models does not capture logical necessity in any significant sense while only amounts to truth under every reinterpretation of the non-logical vocabulary against the fixed actual world.

The prevailing view in literature treats this challenge as targeting solely the ability of model-theoretic account to capture the modal significance of our common conception of logic. According to Etchemendy, since necessity is widely considered as (one of) the most significant component(s) of our pre-theoretic conception of logic, Tarski's model-theoretic account fails to be an adequate analysis of this notion, much less deserving the near-universal acceptance it has received.

[3] For example, Hjortland O. T. (2019) explores how logical theories are justified under anti-exceptionalism and analytic claims are part of what is being questioned.

There are several interpretive statements this paper aims to make. First, I argue that the above reading overlooks a deeper philosophical significance of Etchemendy's distinction, which could shed new light on a correct appraisal of Etchemendy's attack. Intuitively, the distinction between interpretational and representational semantics nicely corresponds to the distinction between the analytic and synthetic nature of logic[4], for both turn on whether logical validity or logical truth is ultimately grounded in meaning or in facts about the world (or both). If logic is analytic, then a logical truth p is true in virtue of the meaning of its constituent expressions. If, on the other hand, the model-theoretic account is interpretational, then models merely assign meanings to otherwise uninterpreted formulas or sub-sentential expressions, and logical validity is explained as generalized truth under every reinterpretation of the non-logical vocabulary while the world remains fixed. This, at first sight, just aligns with what the analyticity of logic dictates[5]. Conversely, if the model-theoretic account is representational, then models represent possible configurations of the world. Under this view, validity is achieved through the interaction of two distinct components: the fixed meaning of the formula and the structure of the varied possible worlds. Consequently, logical validity does not depend solely on meaning, the world also plays an indispensable role, which aligns naturally with the claim that logic is synthetic.

Viewed in this light, Etchemendy's criticism acquires a broader significance: since Etchemendy's major attack on interpretational semantics is that it cannot help us secure logical necessity, and there is this alignment between interpretational semantics and the analytic nature of logical consequence, then, this attack can seemingly be viewed as the charge that the analytic nature of logic cannot explain logical necessity. This of course refutes philosophers' long-standing claim that logical necessity can be explained by the analyticity of logic.[6]

[4] This is also alluded to by Shapiro (1998, p. 19).

[5] I say "at first sight" because under this characterization, interpretational semantics still differs from the traditional conception of logical analyticity in two important respects. First, the latter attributes logical truth to the meanings of the logical constants, whereas the former concerns only reinterpretations of the non-logical vocabulary. Secondly, interpretational semantics retains an essential dependence on the actual world, since generalized truth is evaluated with respect to the actual world, while the analyticity thesis aims to explain logical truth independently (in some sense) of fact. These divergences will be addressed in section 3.

[6] Again, Frege, Russell and Carnap are on the list, though they endorse this claim for different reasons. Note that we could also add to the list people like Kant, though in Kant's case, it is still

This paper will also discuss the most influential response to Etchemendy, i.e. Sher's formal-structural account of logical consequence. Sher (2016) argues that logical necessity is grounded in formal features of reality, expressed by her "formality thesis". Thus, her account provides a systematic response to Etchemendy by locating the foundation of logic in world's formal structure. I argue that, from the perspective of the analytic/synthetic distinction, Sher's proposal amounts to explaining logical necessity by appealing to the synthetic character of logic.

Beyond these interpretive statements, there will be two central claims: First, I argue that Etchemendy's attack ultimately rests on the claim that interpretational semantics lacks two essential ingredients for securing logical necessity: the legitimacy of evaluating formulas over domains of different cardinalities and of employing cross-term reference. It is precisely the absence of these two elements that motivates his rejection of interpretational semantics as an adequate account of logical consequence. Once this diagnosis is made explicit, however, it becomes clear that Etchemendy's criticism targets only a partial conception of interpretational semantics—one concerned exclusively with the meanings of non-logical constants. I argue instead that a fully developed interpretational semantics must also incorporate an account of the meaning of the logical constants. With this requirement met, interpretational semantics can be understood as a modern model-theoretic formulation of the traditional analyticity thesis and is therefore capable, in principle, of explaining logical necessity.

Second, I suggest that not any account of the meaning of logical constants is sufficient for explaining logical necessity. In particular, I suggest that the explanatory work must be done by their intensional meaning rather than merely by their denotations or semantic clauses. On this view, the legitimacy of evaluating formulas over different domains and the semantic coordination embodied in cross-term reference are not externally imposed constraints but arise from the intensional meaning of the logical constants.

Another key term requiring clarification is necessity. This paper does not presuppose the widely accepted identification of necessary truth with truth in all possible worlds. Instead, for the purposes of the present discussion, it treats logical necessity as, plainly, truth in all models. In doing so, the paper offers only a partial analysis of logical necessity and asks a prior question of how

somewhat controversial whether he indeed regards logic as analytic. See Rey (2023) for a summary of relevant debates.

the model-theoretic notion of truth in all models is to be secured. Thus, it deliberately sets aside broader metaphysical questions of what logical necessity ultimately consists in (no matter if it is metaphysical or epistemic) and what is represented by a model, focusing instead on the semantic conditions under which model-theoretic validity can be achieved.[7]

The plan for this paper is as follows: in section 2) I begin to set our focus on model-theoretic logical consequence, explicating in detail Etchemendy's distinction between interpretational and representational semantics and his attack against Tarski's account. I argue that this distinction can be understood as a reformulation of the traditional analytic/synthetic distinction, albeit one that incorporates a non-standard conception of the analyticity of logic. In section 3) I argue that Etchemendy's attack on interpretational semantics boils down to the two essential ingredients necessary for securing logical necessity, and these two ingredients can be obtained only if we turn to the meaning of logical constants. This also implies that what Etchemendyan interpretational semantics[8] misses is exactly the traditional analytic thesis. In section 4) I explicate Sher's response to Etchemendy's attacks and her foundation for logical consequence. It is shown that her formal-structural account amounts to utilizing the syntheticity of logic to explain logical necessity. in section 5) I suggest that instead of retreating to semantics clauses, a more promising solution is to ground logical necessity in the intensional meaning of logical constants.

2. Etchemendy's Attack on Interpretational Semantics and Logical Analyticity

Though Kant seldomly talks about the analyticity of logic, this thesis gains attraction gradually and became part of the discipline's common lore since the rise of logical empiricism. But not entirely. In a series of paper Hintikka (1965, 1966) argues, drawing upon Kant[9], that first-order logical truths can be analytic

[7] As pointed out by a reviewer, Etchemendy deliberately refrains from adopting a specific understanding of the modal notion in his objection too. Thus, it makes sense for me to endorse the same strategy if I aim to refute his objection.

[8] I use this term to refer to Etchemendy's construal of interpretational semantics.

[9] In this paper I shall not address many of the historical and interpretational questions regarding Kant's distinction between analytic/synthetic and its relation to modern distinction.

in one sense, but synthetic in another.[10] The central idea of the argument is to define the analyticity of logic in relation to a specific feature of argument steps, i.e., if a valid argument step does not introduce any new individuals into the discussions, then the argument step should count as analytic. Viewed in this light, there are arguments in polyadic first-order logic that are synthetic in the defined sense. Interesting thesis as it surely is, this conception is restricted to the proof-theoretic conception of logical consequence. This is unsatisfactory because the present paper is concerned with the model-theoretic conception of logical consequence, whose nature and philosophical foundation deserve investigation independently of its formal relationship with proof theory.[11]

Moreover, when it comes to the more traditional definition of analyticity, which states that sentences are true by sole virtue of the meanings of the terms they contain (define as type I in the following quote), Hintikka dismisses it without hesitation:

> In this paper I shall disregard definitions of type I or I (a). Recent discussion has demonstrated, it seems to me, that they are unsatisfactory as they stand. Moreover, they make logical truths trivially analytic, and are therefore beside my present purpose. (Hintikka 1965, p.179)

I hope to show in the paper that logical truths are not trivially analytic, even if we accept this traditional definition and value logical truths according to it; rather, the analyticity of logic is to be argued for against verdicts of many of the most influential philosophers. Incidentally, by "recent discussions" Hintikka includes Quine's attack on analyticity, but Quine's position is more delicate and somewhat contrary to intuition: for, while Quine, in his celebrated paper "Two dogmas of empiricism", aims to show the unviability of analytic truth, this rejection of analyticity does not extend to logical truths. In fact, he is willing to limit

[10] Unfortunately, not many philosophers notice Hintikka's argument, see Larese's (2023) attempt to draw attention to Hintikka's contribution. Moreover, this conclusion is therefore contrary to Kant and Frege's position and mostly the latter, since Kant's pure general logic is still restricted to Aristotelian syllogism, while Frege's logic is already classic modern logic. And for Frege, laws of logic are analytic because they are chosen as axioms of the system.

[11] One might argue here that since first-order logic is complete and sound, what's the point of going beyond proof-theoretic investigation? The answer is we may want a level of explanation or understanding that is more semantic in nature. This situation is similar to that of using Kreisel's "squeeze argument" to prove logical necessity or using completeness to establish the Lowenheim-Skolem theorem. Although proof-theoretic mathematical results are in these cases useful, they cannot afford a genuine resolution for the problem at issue.

the application of 'analytic' to genuine logical truths in which only logical constants occur essentially.

Now, to turn our attention to model-theoretic apparatus originated from Tarski's work, it is understandable that those who subsume logical truths under analytic truths may find little appeal in the model-theoretic definition of logical consequence. The definition is, simply put, if a formula is valid in the model-theoretic sense, it must be true in all models, or there is no model in which the formula could be false:

$$\vDash \varphi \text{ iff all models model } \varphi$$

And models here are set-theoretic constructs, which are in turn based on axioms like ZFC. In contrast, to claim that logical truth is analytic is to understand logical truth as true solely in virtue of the meaning of expressions contained, and this is surely a far cry from existence or non-existence of any set-theoretic structures yielded by the model-theoretic definition.

Thus, to determine whether model-theoretic logic can nevertheless be analytic, we must examine how models can be understood as linking to the meaning of expressions. This motivates us to turn to Etchemendy[12]. In his book *On the concept of logical consequence*, he draws the distinction between interpretational and representational semantics, two completely different roles that model-theoretic structures can play in determining model-theoretic validity, it turns out that interpretational semantics could correlate to the analyticity of logical truths.

The quickest way to get familiar with the distinction is through truth tables. For instance:

Snow is white	Snow is not white	Snow is white or not white
T	F	T
F	T	T

[12] As will become clear later, I do not mean to suggest that Etchemendy himself has any intention of drawing a connection between model-theoretic logical consequence and analyticity.

Not only are these kinds of truth tables useful for specifying the meaning of logical constants, but they also provide us with tools to judge why a sentence like 'snow is white or not white' is logically true. The sign is that in all the models that are listed below the first row, the sentence turns out to be true eventually. What is the role played by a model? The unsophisticated answer is that a model just assigns a truth-value to the atomic sentence, and all models would exhaust the range of truth-values that the atomic sentence could possibly have. To illustrate, in the above table the first model, which is also the second row, assigns truth to the sentence 'snow is white', and the second model assigns falsity to the sentence 'snow is white'.

Here, the key insight of Etchemendy is that the distinction between interpretational and representational semantics arises from the question: "but what is exactly modelled by each row?" His answer is this: while under the understanding of interpretational semantics, the second model (or the third row) changes what the sentence 'snow is white' means—probably to *snow is green* (or to any other proposition which is false in the actual world) —while keeping the world configurations constant, thus this sentence is false in the model since in the actual world snow is not green; under the understanding of representational semantics, the second model has nothing to do with meaning alteration, rather 'snow is white' continues to mean *snow is white*, and what alters is the world configuration that sentences should be valued against. To be more specific, since 'snow is white' turns out to be false, the world could be a place where snow is green. The presupposition of this distinction between interpretational and representational distinction is that there are two and only two ways of changes which are modelled by models or rows, and because we can only choose one of these two ways, model change either represents meaning change or world change, but never both.[13]

We can now establish a preliminary correspondence between the interpretational/representational semantics distinction and the analytic/synthetic distinction. Interpretational semantics corresponds to the analyticity of logic because under this semantics models assign meanings to formulas by assigning set-theoretic constructions to every non-logical constant, validity under this semantics thus amounts to quantification over all possible meanings assignable to all

[13] It is thus not hard to notice the basis of the distinction between the two semantics aligns with the essence of correspondence theory of truth, i.e., there are two essential components that makes a sentence true: meaning and fact.

non-logical constants. Since no world or fact plays any substantive role in this conception, interpretational validity aligns with the analytic nature of logic.

Conversely, representational semantics corresponds to the syntheticity of logic. That is because under representational semantics the meaning is given and kept fixed, and models represent configurations of the world. We often assume that models represent all and only possible ways the world could be, which implies that if there is any situation that could render the proposition false, then we have a model to represent it. Under this representational route, a sentence 'p' is logically true by virtue of the joint work of its meaning and the world, which is tantamount to claiming that 'p' is synthetic.

Now, the crucial question is, which semantics best describes Tarski's model-theoretic definition? Etchemendy's answer is interpretational semantics. According to him, in Tarski's original papers from the 1930s, the domain of discourse and the interpretations of non-logical terms were intended to correspond to the actual objects in the world.[14] The models, therefore, range only over the actual universe. Thus, when Tarski says "in every model in which the premises are true...," what he effectively means is "under every possible interpretation of the non-logical terms within the actual world where the premises are true." In other words, the variation among models concerns differences in meaning assignment exhibitable against the background of the actual world, not differences between possible worlds.

Of course, this is not how the story ends. Etchemendy's intention is not to celebrate this distinction, but to expose and criticize it. His principal objection concerns one of the most important features of our pre-theoretic conception of logical consequence, namely its strong modal force, commonly understood in terms of counterfactual circumstances or possible worlds.[15] Simply put, logical inference does not concern itself with material truth-preservation from premises to conclusion; rather, it requires that the conclusion must be true provided the premises are true. I will briefly outline the two interconnected aspects of Etchemendy's criticism.

[14] Whether Tarski assumes a fixed domain in his early account has generated a substantial body of exegetical discussion. (See Corcoran and Saguillo 2011 for an extensive exegetical discussion) For purposes of this paper, I shall set these discussions aside and only focus on the question of whether Tarski's account is compatible with appealing to non-fixed domains.

[15] In fact, Etchemendy's argument largely abstracts from the specific understanding of modality. We can see from below that while his examples demonstrate compatibility with a possible-worlds framework, they do not preclude alternative conceptions.

The first is the so-called "Tarski's fallacy", arguably the culmination of Etchemendy's criticism. Tarski's aim is to establish that if (S) is a Tarskian consequence of (K), then (S) is a logical consequence of (K) in the ordinary, pretheoretic sense, namely necessarily (K→S). Let (P) abbreviate the claim that (S) is a Tarskian consequence of (K). What Tarski needs to establish is therefore: P→ necessarily (K → S).

Etchemendy reconstructs Tarski's proof as follows. Firstly, we suppose that some < K, S > is a logical consequence in the sense of conforming to Tarski's definition, then suppose that all the sentences in K are true but S is false, and see whether we can derive a contradiction. Now, a contradiction is straightforward. For if < K, S > is a logical consequence in Tarski's sense and K′, S′ are the corresponding logical forms of the original sentences (under some choice of logical constants), then all models for K′ would be a model for S′. Now, if we suppose that all the sentences in K are all true but S is actually false, then we would immediately have a model for K′ but not for S′. Hence contradiction.

According to Etchemendy, however, this argument establishes only

$$\text{necessarily } (P \rightarrow (K \rightarrow S))$$

Tarski therefore commits a modal fallacy, or what Etchemendy calls an "illicit shift in the position of the modality" (1990, p. 87).

Closely related to "Tarski's fallacy" is what Etchemendy regards as the central defect of Tarski's account: the reduction principle. In its original form, it states that if a universally quantified sentence is true, then all of its instances are logically true. Etchemendy's point is that, on the interpretational conception of models, model-theoretic conception of validity does not capture the original notion of logical truth. Instead, it explains logical truth entirely in terms of ordinary truth plus universal quantification over interpretations. If this reduction is successful, then its attraction lies in replacing our complicated notion of logical truth—with its modal notion of necessity—with the mathematically tractable notion of universal truth. But this strategy fails because it reduces logical truth to contingent facts about the actual world. A few examples shall better illustrate Etchemendy's point.

The first example is: (adapted from Etchemendy 1990, chap. 8)

If Leslie was president, then Leslie was a man.

Formalized as

$$\forall x \text{ (If } x \text{ was president, then } x \text{ was a man).}$$

According to the reduction principle, the sentence is logically true. But note that this is only because, contingently, every U.S. president has been male in the history of the actual world, implying that the logical status of the sentence depends on an extralogical historical fact.

One might reply that this case is due to the fact that wrong expressions have been treated as logical constants. Acknowledging this, Etchemendy therefore considers a modified reduction principle, according to which only commonly received logical constants (in first-order logic) may remain fixed. However, the modified principle also fails according to Etchemendy. Consider the central recalcitrant case

$$\exists x \exists y \neg (x = y)$$

Since this formula contains no non-logical constants, the modified principle counts it as logically true since it is true in the actual world. But its truth still depends on the contingent fact that there are at least two objects in the actual world. Logical truth therefore remains hostage to extralogical facts, this time concerning the cardinality of the world rather than its history.

Etchemendy develops further examples, but these suffice to reveal the underlying point. The criticism of the reduction principle follows the same general line of reasoning as "Tarski's fallacy", although the two target different aspects of Tarski's account. "Tarski's fallacy" argues that Tarski proves only

$$\text{necessarily } (P \rightarrow (K \rightarrow S))$$

instead of

$$P \rightarrow \text{necessarily } (K \rightarrow S).$$

The reduction principle explains why this gap cannot be bridged. According to Etchemendy, Tarski's semantic definition, following the spirit of interpretational semantics, establishes only generalized truth under reinterpretations in the actual world. A universally quantified sentence may indeed be true in the actual world while some of its instances nevertheless fail to be logically necessary. In other words, the reduction principle concerns what P means or implies. It says:

P expresses only generalized truth.

While the modal fallacy concerns what inference Tarski makes from P. It says:

From a thesis about generalized truth, Tarski infers about logical necessity.

Thus, "Tarski's fallacy" exposes the formal mismatch between what Tarski's proof establishes and what it is intended to establish, while the critique of the reduction principle identifies the source of that mismatch.

Now, drawing connection to the preceding distinction between the two semantics, it can be demonstrated that Etchemendy's critique possesses a broader philosophical significance than is commonly acknowledged. I have argued that the interpretational understanding of models corresponds to the thesis that logic is analytic. Consequently, if Etchemendy is right that interpretational semantics cannot account for logical necessity, his objection should also be understood as a challenge to the claim that logical necessity can be explained by the analyticity of logic.[16]

But this conclusion is premature. For there is still a gap between interpretational semantics and the common or the traditional conception of the analytic nature of logic. This also exhibits an important ambiguity involved in the analyticity thesis exposited so far. Throughout the preceding discussion, interpretational semantics has been understood as varying only the meanings of non-logical expressions. If we derive the analytic nature of logic from this alone, then logic would be analytic in terms of the meanings of non-logical constants or, more precisely, quantification of all the possible meanings assignable to non-logical constants. Yet the traditional claim that logic is analytic concerns not meaning in general but rather meaning of a specific kind: the meaning of the logical constants. Therefore, whether Etchemendy's criticism can also be targeted at the analyticity thesis depends on whose meaning the analyticity thesis intends to invoke. Since, traditionally, it is the meaning of logical constants that has played a prominent role in deriving the analytic nature of logic, we are not yet in a position to adjudicate that Etchemendy's criticism amounts to a genuine attack on the analytic nature of logical consequence.

Other than the above gap, there is another mismatch between interpretational semantics and the traditional analyticity thesis. Note that according to Etchemendy, interpretational semantics still requires us to evaluate the inter-

[16] This line of thought has been influential throughout the history of philosophy. E.g., Logical positivists held that logical and mathematical truths are necessary because they are analytic, that is, true solely in virtue of meaning and therefore independent of how the world happens to be. While Kant argues that analytic judgments are a priori since their truth follows from conceptual analysis alone rather than experience; and precisely because they are a priori, they are necessarily true.

preted sentence against the actual world, therefore the world or fact still possesses a significant role in determining logical validity under this approach. However, this is not so under logic's analyticity thesis, since this thesis demands that those logical truths are true solely in virtue of the meaning of logical constants, and no world or fact is supposed to have an explanatory power regarding their truth-status.

In the next section, I shall give a more thorough analysis of what is lacking in Etchemendyan interpretational semantics. In particular, I argue that the two ingredients required for securing logical necessity (truth in all models)—the legitimacy of evaluating formulas over different domains and the possibility of cross-term reference—can be grounded in the meanings of logical constants. Therefore, once we shift our focus from the meaning of non-logical constants to that of logical constants, interpretational semantics could possess the very resources that Etchemendy takes to be missing. This significance of this substantive move consists not only in helping us blocking Etchemendy's criticism per se, it is foreseeable that it could also help us diminish the aforementioned gap and mismatch between interpretational semantics and the traditional analyticity thesis: the analyticity of logic is no longer understood merely in terms of varying the meanings of non-logical expressions, and the explanatory burden of logical truth is shifted from facts about the world to the meaning of the logical constants itself.

3. What Interpretational Semantics Requires for Explaining Necessity

Modern model-theoretic apparatus seems to have an answer to Etchemendy's criticism. By allowing arbitrary domains, we can appeal to a model with a single object in its domain and thereby rendering $\exists x \exists y \neg (x = y)$ false. Yet this move appears to fall outside Etchemendyan interpretational semantics, because, according to the reduction principle, the spirit (and merit) of interpretational semantics consists in its reducing logical truth to generalized truth in the actual world. Since there are no variations of possible worlds under interpretational semantics, we are therefore deprived of the ground of invoking domains of different cardinalities, or so argues Etchemendy.

The difficulty, then, is not merely that interpretational semantics lacks sufficiently many models. Rather, the deeper question is what unifies models with different domains into a single semantic framework. Under representational route, the principled reason is that models represent possible ways the

world could be. That is, not only must there be a model for every possible state of the world, but the entire class of models must collectively exhaust all possible ways the world could be. By contrast, under interpretational semantics we lack such principled reason, for models only function to assign meanings to otherwise uninterpreted formulas or sub-formula expressions. Surely, we could still say that different models amount to varied interpretations of non-logical expressions, however, as we have seen from the central case proposed by Etchemendy, the crucial variation at issue is the cardinality of domains, and this is a settled issue if it is always the objects in the actual world that occupy the domain of all models that we can invoke.[17] Therefore, unless there is a principled reason for regarding such models as interpretations unified, truth across all models amounts to nothing more than truth across an arbitrary collection of semantic structures of different cardinalities. In that case, it would be difficult to see why validity across those models should amount to (or at least ground) logical necessity rather than merely accidental generalization.

The crucial question raised by Etchemendy's criticism, then, is this: what could provide the required unity across different models? If interpretational semantics is to explain logical necessity, there must be some semantic element that remains fixed while the interpretations of non-logical constants vary. Otherwise, there is no principled reason why models with different domains should count as unrelated semantic structures.

This, however, is only one part of the problem. Even if we could explain why different models belong to a single semantic framework, we would still need to explain why the semantic values assigned within any individual model constitute a coherent interpretation rather than an arbitrary collection of assignments. This has to do with an important mechanism Etchemendy himself identifies, namely what he calls "cross-term reference". (1990, chap. 5) The idea of cross-term reference is that the interpretations assigned to different expressions are not independent but should satisfy certain semantic relations to one another in a single model. A prominent illustration of cross-term reference is that, when a quantified sentence is interpreted by a model, a term must be assigned an object drawn within the domain assigned to the quantifier. Thus, the interpretations of quantifiers, variables and predicates are systematically coordinated rather than arbitrarily assembled in a single model.

[17] This is not to say that determining the cardinality of the entities in the actual world is an easy matter. However, for the sake of Etchemendy's argument, it suffices that this cardinality can in principle be settled.

For example, when we use a model to interpret the formula $(\forall x)(Bx \vee \sim Bx)$, and if the domain of the universal quantifier is set as D, then we should assign to the predicate B subsets of D. The semantic values assigned to the different expressions are therefore not independent but systematically coordinated. Etchemendy does not take this as motivated under interpretational semantics because under his construal interpretations of different expressions should not stand in some fixed relations to one another, while the only legitimate restrictions are those imposed by semantic categories. As Etchemendy remarks: (1990, p. 68)

In the present semantics, certain d-sequences are excluded from the class of models not because they suggest a semantically ill-behaved interpretation of any individual expression, but because the interpretations of different expressions fail to stand in some fixed relation to one another.

(note that Etchemendy's 'd-sequence' can just be identified with the modern conception of model)

Etchemendy's criticism therefore exposes two explanatory tasks that any interpretational account of logical consequence must accomplish. First, it must explain why formulas may legitimately be evaluated over a unified class of models whose domains differ in cardinality. Secondly, it must explain why the semantic values assigned within each model are coordinated. Without the former, truth across all models lacks modal significance; without the latter, an individual model ceases to be a genuine interpretation and becomes merely an arbitrary assignment of semantic values.

This diagnosis also illuminates what is required for logic's analyticity thesis to vindicate interpretational semantics. Now, if the analyticity thesis merely draws upon reinterpretations of the non-logical vocabulary, then this is merely a repetition of our statement already under discussion and contributes nothing towards explaining logical necessity. For such a thesis merely tells us that logical truths remain true under reinterpretations of the non-logical expressions; it does not explain why those reinterpretations form a unified semantic framework or why each interpretation possesses the internal semantic coordination.

Therefore, I propose that we should appeal to the alternative traditional understanding of the analyticity thesis. That is, logical truths are analytic in the sense that they are true in virtue of the meaning of logical constants. This is going to help us obtain the ground for invoking the two essential elements for ex-

plaining necessity. On this view, the meaning of the logical constants itself functions to unify the semantic structure required for logical consequence. More specifically, the meaning of the logical constants plays two related but distinct semantic roles. Across different models, it remains invariant and makes different models interpretations of the same language. Within each individual model, it determines how the semantic values assigned to different expressions are systematically related. Therefore, the proposal implies that when the analytic nature of logic arises from the meaning of logical constants, this analyticity is not opposed to logical necessity; rather, it is precisely what explains why logical consequence possesses its distinctive modal force.

Interestingly, this is a point that Etchemendy himself implicitly expresses in his "Reflections on Consequence", published almost twenty years after his harsh criticism of interpretational semantics. In the paper, after realizing that there are inferences like modus ponens whose validity is by no means corrigible and thus qualifies as genuine, he says: "My own view is that we recognize this by virtue of the meaning of the expression if ... then and our knowledge of how the remaining constituents can contribute to the truth values of the premises and conclusion." (p. 268) Although Etchemendy continues to argue that many alleged logical truths ultimately depend upon brute facts about the world, Etchemendy's late-come concession is nevertheless still significant because it demonstrates that appealing to the meaning of logical constants could be exactly the right direction.

The above proposal suggests that turning to the meaning of logical constants can bridge the mismatch between interpretational semantics and the traditional analytic nature of logic. Furthermore, this approach addresses the mismatch between them regarding logical truth's residual dependence on contingent facts. In fact, notice that in all the counterexamples that Etchemendy mounts against interpretational semantics, no logical truth is appreciated as being true solely in virtue of the meaning of logical constants, rather, there is always an element of fact (or an obtained state of affairs) at play. No wonder that counterfactual cases can always be generated to show that the logical status of generalized sentences hinges upon contingent facts about the world.

Now, the traditional analyticity thesis holds that logical truths are true solely in virtue of the meanings of the logical constants. This immediately raises a philosophical question. Notice that even if a logical truth is true in virtue of meaning alone, it is still a truth. And under a broadly correspondence-theoretic conception of truth, for a sentence to be true two conditions must obtain: the

sentence must express a determinate proposition, and the world must be as that proposition says it to be. For example, even if the tautology $p \vee \neg p$ is true in virtue of the meanings of disjunction and negation, it is nevertheless true because, whatever proposition $p \vee \neg p$ expresses, it is a fact or an obtain states of affairs that makes the sentence true. Truth therefore still presupposes the participation of the world, even in the case of logical truths.[18]

The crucial question, however, is whether the world's participation is merely a background condition for truth or whether it contributes to the explanation of why logical truths are true. The traditional analyticity thesis demands the former: although the world must cooperate in any instance of truth, the explanatory burden falls entirely on the meanings of the logical constants. [19] By contrast, Etchemendy's criticism may be understood as denying precisely this possibility. If explaining logical truth inevitably requires appealing to contingent features of the actual world, then logical truth cannot be explained solely by meaning. How could we respond to Etchemendy's criticism?

Now, since we have specified what is needed for interpretational semantics to explain logical necessity, we can motivate the traditional analyticity thesis as follows: once the semantic framework itself—including the class of models and the semantic relations embodied in cross-term reference—is determined by the meanings of the logical constants, the appeal to all models no longer derives from facts about the actual world but from the semantic roles of the logical constants themselves.

Therefore, what Etchemendy fails to recognize is that even under the general framework of correspondence theory of truth—a framework which he implicitly[20] and I explicitly endorse—we can still plausibly say that logical truth is true solely in virtue of the meaning of logical constants. Under the model-theoretic conception of logical consequence, this claim should not be understood as denying the world's participation in truth. Rather, it means that the meanings

[18] Another example is from Harman (1973, p. 2), as he remarks that even "Copper is copper" depends in part on a general feature of the way the world is, namely, everything is self-identical.

[19] To use an analogy proposed by Russell (2014), in multiplication, the product normally depends on both arguments (e.g., $5\times3=15$). But when one argument is 0, the result 0 is completely determined by that argument alone—the other argument becomes explanatorily irrelevant. The same could apply to the explanatory power of the meaning of logical constants.

[20] We can find in Etchemendy (1990) a sentence confirming this stance: "It would mean they were either true by virtue of meaning alone, or true by virtue of the facts about the world. But of course that is true of any sentence whatsoever." (p. 106)

of the logical constants provide the semantic apparatus required for logical validity and logical necessity—namely, the legitimacy of evaluating formulas over different domains together with the semantic coordination embodied in cross-term reference.

The discussion above merely aims to describe the merit of the proposal. Whether this proposal ultimately succeeds depends upon developing a substantive theory of the meaning of the logical constants, a topic that is both difficult and controversial. While a full treatment is beyond the scope of this paper, I will first examine one leading account: Sher's formal-structural account.

4. Sher's Criticism and the Formal-Structural account

Among the many responses to Etchemendy's critique, Sher's account (1996, 2016) is particularly significant because it aims not merely to answer his objections but to provide a comprehensive foundation for logical consequence. Faced with Etchemendy's criticism of interpretational semantics, Sher has two responses. On the one hand, she joins Etchemendy's camp in criticizing that interpretational semantics only allows for models which share a common predetermined universe—the actual universe—thus falling short of accounting for modality[21]; on the other hand, she claims that the distinction between interpretational and representational semantics does not exhaust all the options that we have. Let's get clearer about these two aspects in turn.

As argued by Sher, interpretational semantics is defined within a material meta-theory, "a theory of 'the actual world'" (1996, p. 666), which means that the domain at our disposal is confined to the collection of actual individuals only. This is the same with Etchemendy's criticism of interpretational semantics, that is, interpretational semantics reduces logical truths to material truths through the so-called reduction principles, a project that barely seems promising.

To overcome this limitation, Sher develops what she calls the "formal-structure" view. According to this view, "a (logical) model represents a formally possible structure of objects relative to the primitive terms of a given language"

[21] Thus, Sher's criticism follows the line of the reduction principle, namely that reducing logical validity to generalized truth involving the actual world fails to account for the distinctive modal force of logical consequence. (see especially Sher 1996, pp. 664-667) Whether analogous considerations apply to epistemic conceptions of necessity is a separate question that lies beyond the scope of this paper.

(1996, p. 675). Two features of this proposal deserve particular emphasis. First, Sher insists that the formal-structural view should not be identified with representational semantics. Although both reject interpretational semantics, they understand models in fundamentally different ways. The second is that it provides a foundation for logical consequence because all the features of logic can allegedly be grounded in the formal structures of the world.

It is perfectly understandable that Sher does not want models to represent possible worlds, for while the former notion is a highly technical notion built upon set-theoretic axioms, the latter notion respects general metaphysical considerations. Consequently, these two frameworks necessarily yield different extensions when predicting what is possible. This is not to mention that possible world's exact extension is still hotly debated because it involves multiple controversial metaphysical notions like possibility, necessity, essence and identity etc. As explicated by Sher (1996), there will be a bunch of inscrutable questions regarding taking models to represent possible worlds, and they are vividly represented as follows:

Consider a discourse about a certain object, say my dog Cerberus. What are all the possible variations in the world vis-a-vis Cerberus? Clearly, there are models in which Cerberus is your dog rather than mine, but is there a model in which Cerberus is human? a model in which Cerberus is another dog? a model in which Cerberus evolved from an egg fertilized by a spermatozoon with an X chromosome... (p. 660)

To avoid these problems, Sher extends the original notion of metaphysical possibility to formal possibility, a notion which is wider in scope. These formally possible structures are just represented by models as set-theoretic constructions: "On my conception, formal possibility is reduced to mathematical existence and formal necessity to mathematical generality" (ibid., p. 682). On this view, models do not represent mysterious alternative possible worlds but formally permissible configurations of objects. Consequently, the domains of models remain constrained by objective facts about reality while simultaneously representing a wider range of possibilities than either physical or metaphysical possibility. Logical possibility is thereby given a precise mathematical interpretation in terms of formal possibility, which is itself grounded in structural invariance of formal properties that logical constants denote.[22]

[22] It is controversial whether Sher's view could really keep a distance from representational semantics. As argued by Sagi (2014), Sher's formally possible structure of objects are just interpretations of the non-logical terms in a possible world. (cf. Shapiro 1998)

This leads us to the most demanding part of Sher's account, that is her understanding of logical formality and attributing to it a significant foundational role. According to this view, the essential feature of logical constants is that they denote those worldly properties which do not "pay attention" to any difference between identities of objects and thus can be demarcated by the criterion of invariance under isomorphism[23]. For example, while the empirical property 'is a dog' would change its extension if we map a set of dogs onto a set of numbers, the formal (or the logical) relation of 'identity' and the property of 'cardinality' remain entirely unaffected. Formal terms do not care which specific individuals exist, only how they are structured. Logical constants are therefore distinguished precisely by their structural invariance.

Moreover, Sher argues that these formal properties are governed by objective formal laws. Hence the "formality thesis":

> Logic (logical consequence) is grounded in formal laws governing reality—laws governing formal features of objects and properties—and it is the broad applicability and strong modal force of such laws that underlies (and explains) the generality and strong modal force of logic. (2016, p. 272)

Thus, together with these formal laws, logical constants are not only entities which lie at the core of logical formality, they also give rise to almost all other features of logical consequence and logical truths, among which are necessity, generality, topic-neutrality, normativity etc. (see Sher 2016, chap. 10).

Now, our discussion in the previous sections provides a new perspective through which we can understand Sher's strategy. It turns out that Sher's view can be seen as attempting to secure the legitimacy of evaluating formulas over different domains and cross-term reference. Let me explain.

Firstly, note that 'all models' is a requisite component of Sher's account. On Sher's account, logical consequence is not concerned with truth-preservation under arbitrary reinterpretations, nor merely with truth-preservation across metaphysically possible worlds, but with truth-preservation throughout the totality of formally possible structures. This point explains why Sher insists that truth must be evaluated with respect to all models. Since logical consequence requires us to determine whether a sentence remains true across all formally possible structures, it is required that each model represents one formally

[23] Crudely, two structures of objects are isomorphic if and only if there is a one-to-one correspondence between their domains such that all formal relations and operations are preserved under the mapping. Nothing in this paper will be affected by any detail of spelling out this criterion.

possible structure of objects, and the class of all models exhausts the range of all possibilities. As Sher writes,

> it is essential that the denotations of both logical and non-logical constants be defined over models, and indeed all models, so that every formally possible state of affairs will be taken into account in determining logical truths and consequences. (2016, p. 282)

This means that restricting ourselves to the actual domain or to any predetermined universe would leave many formally possible structures unrepresented, thereby undermining the modal force characteristic of logical consequence. Consequently, domains of different cardinalities are not introduced as an ad hoc technical device. Rather, they should be invoked because different formally possible structures may contain different numbers of objects, and a sufficiently varied class of models must be able to represent every such structure.

Moreover, the appeal to all models is also necessitated by Sher's account of logical constants. On this account, in devising a semantic apparatus for identifying logical consequence, logical constants play the role of representing formal features of objects that the system is going to take into consideration. Consequently, logical constants' denotations must themselves be determined independently of any particular model. As Sher writes, "the denotations of the logical constants [must] be determined in advance for all models" and remain "fixed throughout the totality of models." (2016, p. 282) Accordingly, logical constants are "semantically defined outside models" (ibid., p. 282), or "are defined by fixed functions over models" (1996, p. 675), whereas non-logical constants receive their interpretations only within individual models. Therefore, on Sher's account, fixed meanings of the logical constants require a settled notion of all models, while the totality of models represents the complete range of formally possible structures relevant to logical consequence.

Secondly, Sher's semantic framework also manages to explain cross-term reference. As we have seen, according to Etchemendy, such coordination cannot simply be assumed, because interpretational semantics is supposed to merely assign meanings to individual expressions according to their semantic categories. While Sher's account provides precisely the missing explanation. The key lies exactly in how the logical or formal properties are defined. As Sher writes,

> Since the logical constants denote formal properties of the denotations of the non-logical constants, their level is usually higher than that of the non-logical

constants (serving as their arguments). (2016, p. 281)

This remark is philosophically significant. It indicates that logical constants are not interpreted alongside predicates and names as independent pieces of vocabulary. Rather, they operate on the semantic values assigned or assignable to non-logical expressions.

For instance, let's say structure S is generally specifiable as a (n+1)-tuple

$S=\langle A,\beta 1,...,\beta n\rangle$,

Now, A is just the domain of the structure, while β_i is either a member of A or a set-theoretic construct of members of A. Then Logical constants are just defined as denoting formal properties or operators operating over such structures in the form of S. Consequently, the arguments of logical properties or operators are not isolated semantic values but the components of a single structured interpretation. A predicate, for example, must denote a subset of the domain, variables must range over members of the domain, and relations must be defined on that domain. These are not independent stipulations attached to individual expressions; they are conditions imposed by the very semantic definitions of the logical constants. Put differently, only if a collection of semantic values forms the kind of structure on which the logical constants are defined, can they count as a legitimate model.

Since these semantic definitions are fixed across all models, the same structural relations are preserved throughout the entire class of models. In this way, the semantic values assigned to terms, predicates, and domains cannot vary independently. Rather, their coordination follows from the fixed meanings of the logical constants themselves. It is precisely this systematic coordination that corresponds to what Etchemendy calls cross-term reference.

Now, a comprehensive evaluation of Sher's view falls outside the scope of this paper. Remember that the main purpose of this paper is rather just to reconsider the analytic/synthetic divide through Etchemendy's interpretational/representational distinction—more specifically, to clarify exactly how interpretational semantics is linked to the analytic nature of logical consequence—hence what is more pertinent here is to determine which strategy Sher's account ultimately sides with.

The preceding analysis demonstrates that Sher's position serves to legitimize the evaluation of formulas across diverse domains, as well as cross-term reference, which we have identified as essential for explaining logical necessity

under interpretational understanding of models. However, because Sher's formal-structural view ultimately locates logical constants as denoting terms—they denote worldly properties that are isomorphism invariant—it differs vastly from appealing to the analytic nature of logic. Instead, we are led to the following way of explaining logical necessity: because of the maximal isomorphism invariance feature of logical properties that we can gain independent grasp of, logic applies universally between any actual and counterfactual isomorphic structures, while the former explains the generality of logic, the latter explains the necessity of logic. (2016, pp. 290-291)

Therefore, although in a broad way we can still say that Sher bases her explanation of logical necessity upon the meaning of logical constants, it is ultimately the syntheticity of logic that explains logical necessity. This is connected to the grander commitment of Sher's philosophical commitment. For Sher aims to show that logic is not grounded only in the mind, but also substantially grounded in the world. The phrase 'in the mind' serves as an umbrella term for historical positions that seek to ground logic exclusively in the rules of language, psychological constitution, or semantic meaning etc. These theses are undermined by Sher's account, since on this account logical constants ultimately denote objective and formal properties in the world, and it is these properties that provide the ground for explaining logical necessity.

Hence, although Sher's strategy aims to shift the explanatory burden of logical necessity to logical constants, it does not preserve the interpretational conception of logical consequence, let alone the traditional analyticity thesis. If these are to be preserved, which is exactly the goal of this paper, a different conception of the meaning of logical constants is required.

5. Towards an Analytic Foundation for Logical Necessity

Our discussion in the previous sections points to what I take to be the central omission in Etchemendy's construal of interpretational semantics: there is no explicit account of the meaning of logical constants. In fact, as we have shown in section 2, either the choice of logical constants is assumed to be a practical matter under his construal of interpretational semantics, or the demarcation of logical constants is randomly set to conform to classical first-order logic. This is understandable because Etchemendy's goal is to show that, no matter how the line between logical and non-logical is to be drawn, we are unavoidably led to

"Tarski's fallacy", thus we do not have to care for the meaning of logical constants. Consequently, he has little reason to investigate the meaning of the logical constants.

Once our concern shifts from criticizing Tarski to explaining logical necessity, however, this omission becomes decisive. To provide a full-blooded account of logical constants, we should not only provide a principled demarcation of logical constants but also seek to account for their distinctive meaning as well as how it contributes to logical consequence. Note that it is exactly this element that remains stable across different models. Therefore, it is presumably this missing piece that could unify different models and thereby make them relevant to one another under a single unified semantic framework. Concerning cross-term reference, Etchemendy also ignores the possibility that the distinctive meaning of the logical constants—which remains fixed across different models and presumably makes those models relevant—could provide us with a principled reason to impose cross-term reference between interpretations of logical and non-logical constants.

We have also seen from section 4 that Sher's account gives a very demanding interpretation of what it means for logical constants to remain stable across different models, i.e. they are assigned with prefixed denotations in all models, which are supposed to represent all formally possible structures of objects. Unwilling to abandon logical analyticity, we must ask whether an alternative route exists for explaining logical necessity, and whether the requisite semantic apparatus can instead be grounded within interpretational semantics augmented with the analyticity thesis.

In this section, I would like to gesture toward what I take to be a more promising direction for answering this question. The general strategy is to retain the analyticity of logic while abandoning a denotational conception of the meanings of logical constants.

As a first pass, a natural step in this direction is to focus on the semantic contribution of logical constants rather than on the worldly entities they denote. Within a model-theoretic framework, this amounts to taking the semantic clauses associated with the logical constants as constituting an essential part of (or even identical to) their meaning.[24]

[24] Sometimes, it is regarded as a merit of model-theoretic semantics that we do not have to investigate the metaphysical commitment of these semantic clauses.

This kind of step can be found in Sánchez-Miguel's (1993) work. Like the present paper, he argues that Etchemendy criticizes only a partial semantic story and that what is missing is an account of the meaning of the logical constants specifiable by the semantic clauses. His account therefore provides independent support for the diagnosis defended in this paper. After presenting his account, I shall argue that it leaves unanswered the further fundamental question: what is it about the meaning of the logical constants that enables it to secure these semantic resources?

Sánchez-Miguel argues the meaning of logical constants inherently determines the class of domains to quantify over and justifies the use of cross-term reference. (See especially his 1993 section. 3) Significantly, he explicitly connects this capacity of the meaning of logical constants to the analytic nature of logical consequence. Thus, we are led to the following analyticity thesis:

> ... if, for every possible truth value that a sentence could have (that is, *in every possible preformal model),* our original sentence is going to be true (as it is in this case), we can be sure that it is true only in virtue of the meanings of the two logical expressions. (1993, p. 116)

(note that a preformal model on his account could be understood as the combination of all the possible semantic values that non-logical expressions could possess)

This constitutes the core of Sánchez-Miguel's position. To illustrate using Sánchez-Miguel's own example, consider the formula 'p or not p'. As argued by Sánchez-Miguel, the contribution of the two logical expressions to the truth-condition of this formula depends on the truth-value of p, and it does not depend on anything else. This implies that the meaning of the two logical expressions decides the range of possible semantic values. For whether p is true or false exhausts every semantically possible assignment relevant to the evaluation of the formula, and the semantic clauses associated with disjunction and negation determine the truth-value of the whole sentence for each such assignment. In this sense, the meanings of the logical constants themselves, in the form of semantic clauses, bring with them the class of all possible semantic values.

Moreover, the semantic clauses also impose the coordination that Etchemendy calls cross-term reference. The semantic values assignable to the constituent expressions cannot vary independently, since the truth-condition of the complex formula is determined only through the semantic relations established by the logical constants. To be more specific, the meaning of negation

relates the truth-value of ¬p to that assigned to p, while the meaning of disjunction determines how these coordinated truth-values jointly determine the truth-value of the entire formula. Consequently, the semantic relations required for evaluating the formula arise from the meanings of the logical constants themselves rather than from any independently imposed semantic constraints.[25]

Now, critically, how can we come to know that any sentence in the logical form 'p or not p' is true solely in virtue of the meaning of the two logical expressions? Sánchez-Miguel's answer is this: admittedly, in this case we cannot decide the exact truth-value of p just based on the semantic story we give for the two logical expressions involved, for p's truth-value depends on "nonsemantic matters", however, if the complex sentence is true for every possible truth-value that p could have—i.e. 'p or not p' is true no matter p is true or false—then, we can be sure that the sentence in the logical form 'p or not p' is true solely in virtue of the meaning of logical constants. (ibid., p. 116)

If successful, Sánchez-Miguel's proposal restores precisely the connection between analyticity and logical necessity that Etchemendy rejects. More importantly, it supports the central diagnosis defended throughout this paper: the semantic resources required for logical necessity arise from the meaning of the logical constants rather than from reinterpretations of the non-logical vocabulary alone. Nevertheless, it leaves a deeper explanatory question unanswered. Sánchez-Miguel tells us that the meaning of the logical constants determines the relevant class of models and licenses cross-term reference, but he does not explain why the meaning of the logical constants possesses this capacity. Another way to put this is that on the current account the meaning or the semantics of logical constants is taken as primitive. It seems that we are in a position to only accept that logical constants make their contributions to the truth-condition of sentences in the way specified by various semantic clauses (or truth-tables), while why they make precisely those contributions is out of question.[26]

This problem becomes particularly apparent when it comes to Sánchez-Miguel's remarks about the nature of logical constants, as he maintains: "I do

[25] I focus on propositional logic here for ease of exposition. Note that while Sánchez-Miguel (1993, pp.115-117) is not (entirely) explicit on these two points, these argumentations likely represent his position.

[26] The problem of logical pluralism also kicks in at this point. We are told how logical constants contribute to truth-conditions, but not why they contribute in precisely that way (e.g., conforming to classical logic) rather than another (e.g., conforming to non-classic logic). The explanatory burden has therefore been relocated.

not need to explain what it is that makes the logical constants peculiar, that semantic feature which philosophers have tried to capture as their 'contentless' or 'formal' character." (ibid., p. 114) Moreover, as he continues to suggest, his account is compatible with Hacking's explanation of logical constants in terms of inferential rules and the "Tarski-Sher thesis" which takes logical constants as denoting worldly structures.

However, the analyticity thesis endorsed by him clearly does not accept such a liberal attitude. For, if the meaning of logical constants is exhausted by some worldly structures that they denote as required by "Tarski-Sher thesis", then, truth in virtue of the meaning of logical constants amounts to nothing but truth in virtue of fact about the world. This would deprive any sense of our seeking to account for logical consequence as being analytic, and the appeal to analyticity becomes merely verbal. The analyticity thesis therefore requires a richer conception of meaning than mere appealing to semantic clauses.

One natural way of pursuing this further explanation is to distinguish, in a broadly Fregean spirit, between the sense and the reference of the logical constants. Standard model-theoretic semantics primarily characterizes logical constants at the level of reference: their denotations are fixed across all models and thereby contribute to the truth-conditions of complex formulas as they do. While comparatively little attention is given to their sense—the aspect of meaning that explains why those semantic roles and denotations are the ones they are.

One interesting proposal along this line of thought is this: if the sense of a logical constant is understood, for example, in terms of the inferential roles associated with it, then the familiar semantic clauses of model theory no longer appear as primitive. Rather, they become semantic realizations of the inferential roles constituting the senses of the logical constants.[27] On such a view, it is intelligible that the appeal to a unified class of models, together with the semantic coordination embodied in cross-term reference, is licensed by the sense of the logical constants. Logical necessity would then be explained through analyticity, not because logical constants make those contributions to truth-conditions as

[27] For instance, suppose the sense of conjunction is (partly) constituted by its introduction rule,

$$\frac{A \quad B}{A \wedge B}.$$

Then the semantic clause for conjunction must assign to $A \wedge B$ the truth-condition that it is true iff both A and B are true, so that when A and B are true, $A \wedge B$ is also true. In this way, we say that the introduction rule is realized or confirmed.

they do, but because the semantic apparatus required for model-theoretic consequence ultimately derives from the sense of logical constants.

This is connected to Sher's using isomorphism-invariance as a criterion to demarcate logical constants. It turns out that, as an extensional criterion, isomorphism-invariance merely reflects the logical/non-logical distinction grounded in the intensional level, a point endorsed explicitly by Sher herself. According to Sher (2021, p. 3950), "the selectivity of properties that have a distinct extension and intension is determined by their intension", and it is only because the notion of invariance is conceived to be an extensional notion that "to accurately express the selectivity of properties in terms of invariance, we have to 'extensionalize' their intension". The implication of this is not that the extensional criterion of isomorphism-invariance gives us the wrong result, rather, it is that the explanation for why isomorphism-invariance holds for logical constants should be located at the intensional level. And this should remind us that in order to account for the semantic roles logical constants play in model-theoretic semantics, we could appeal to the intension or sense of logical constants as a viable alternative.

Developing such an account further than this point, however, lies beyond the scope of the present paper. It would require a substantive theory of the meaning of logical constants capable of explaining why the familiar semantic clauses—and, more fundamentally, the semantic roles associated with the logical constants—are the ones they are. The goal of this paper is more modest, or even largely interpretational. Throughout the paper, I have argued, first, that Etchemendy's celebrated distinction between interpretational and representational semantics mirrors the traditional distinction between the analytic and the synthetic nature of logical consequence. Secondly, I have argued that Etchemendy's criticism reveals the conditions that any adequate explanation of logical necessity must satisfy under an interpretational conception of logical consequence. Thirdly, I have gestured that while Sher satisfies these conditions by appealing to the formal structure of reality, the same conditions can, in principle, be satisfied by appealing instead to the intensional meaning of the logical constants.

If this proposal is ultimately viable, then the traditional analyticity thesis should not be abandoned in response to Etchemendy's criticism of interpretational semantics; rather, it should serve as a recourse: logical necessity is ultimately grounded not in quantification of all the possible semantic values of non-logical constants, but in a richer, multilayered conception of the meaning of log-

ical constants—one capable of explaining both the legitimacy of evaluating formulas over all domains and the semantic coordination embodied in cross-term reference.

REFERENCES

Boghossian, P. A. (1996). Analyticity reconsidered. *Noûs, 30*(3), 360–391.

Corcoran, J., & Sagüillo, J. M. (2011). The absence of multiple universes of discourse in the 1936 Tarski consequence-definition paper. *History and Philosophy of Logic, 32*(4), 359–374.

Etchemendy, J. (1990). *The concept of logical consequence.* Harvard University Press.

Etchemendy, J. (2008). Reflections on consequence. In D. Patterson (Ed.), *New essays on Tarski and philosophy* (pp. 263–299). Oxford University Press.

Harman, G. (1973). *Thought.* Princeton University Press.

Hintikka, J. (1965). Are logical truths analytic? *The Philosophical Review, 74*(2), 178–203. https://doi.org/10.2307/2183522

Hintikka, J. (1966). An analysis of analyticity. In P. Weingartner (Ed.), *Deskription, Analitizität und Existenz* (pp. 193–214). Pustet.

Hjortland, O. T. (2019). What counts as evidence for a logical theory? *The Australasian Journal of Logic, 16*(7), 250–282.

Larese, C. (2023). Hintikka's conception of syntheticity as the introduction of new individuals. *Synthese, 201*(6), 207.

Quine, W. V. O. (1951). Two dogmas of empiricism. *The Philosophical Review, 60*(1), 20–43.

Rey, G. (2023). The analytic/synthetic distinction. In E. N. Zalta & U. Nodelman (Eds.), *The Stanford Encyclopedia of Philosophy* (Spring 2023 ed.). https://plato.stanford.edu/archives/spr2023/entries/analytic-synthetic/

Russell, G. (2014). Quine on the analytic/synthetic distinction. In G. Harman & E. Lepore (Eds.), *A companion to W. V. O. Quine* (pp. 181–202). Wiley-Blackwell.

Sagi, G. (2014). Models and logical consequence. *Journal of Philosophical Logic, 43*(5), 943–964.

Sánchez-Miguel, M. G. C. (1993). The grounds for the model-theoretic account of the logical properties. *Notre Dame Journal of Formal Logic, 34*(1), 107–131.

Shapiro, S. (1998). Logical consequence: Models and modality. In M. Schirn (Ed.), *The philosophy of mathematics today* (pp. 131–156). Oxford University Press.

Sher, G. (1996). Did Tarski commit "Tarski's fallacy"? *The Journal of Symbolic Logic, 61*(2), 653– 686.

Sher, G. (2016). *Epistemic friction: An essay on knowledge, truth, and logic.* Oxford University Press.

Sher, G. (2021). Invariance as a basis for necessity and laws. *Philosophical Studies, 178*(12), 3945–3974.