Pluralism in Scientific Problem Solving. Why Inconsistency is No Big Deal

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ABSTRACT

Pluralism has many meanings. An assessment of the need for logical pluralism with respect to scientific knowledge requires insights in its domain of application. So first a specific form of epistemic pluralism will be defended. Knowledge turns out a patchwork of knowledge chunks. These serve descriptive as well as evaluative functions, may have competitors within the knowledge system, interact with each other, and display a characteristic dynamics caused by new information as well as by mutual readjustment. Logics play a role in the organization of the chunks, in their applications and in the exchange of information between them. Epistemic pluralism causes a specific form of logical pluralism. Against this background, the occurrence of inconsistencies will be discussed together with required reactions and systematic ways to explicate them. Finally, the place of inconsistencies in the sciences will be considered. Seven theses will be proposed and argued for. The implications of each of these for pluralism will be considered. The general tenet is that paraconsistency plays an important role, bound to become more explicit in the future, but that the occurrence of inconsistencies does not basically affect the need for pluralism.

1. Introduction

It is the aim of this paper to explore links between pluralism and inconsistency toleration in science. There are many forms of pluralism. The form that interests us most in the present context is logical pluralism, roughly that several logics play a role in human reasoning in general and in science in particular.

Allow me to start with a personal comment. I have been a convinced logical pluralist for many years and have defended my position in several papers. Yet,

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reactions to those papers seem to rely on a misunderstanding of my position. This caused some sadness, but especially guilt about the obvious failure to express myself adequately. My present diagnosis is that my position will become clearer if I explain first what I mean by epistemic pluralism—see Section 2— and next, in Section 3, clarify the way in which this influences the intended notion of logical pluralism.

The claim from the title, that inconsistency is no big deal, states the position that the advent of paraconsistent logics did not cause a dramatic change with respect to logical pluralism. I do not mean that the advent of paraconsistent logics was insignificant. Quite to the contrary. Notwithstanding the heavy opposition, even hostility, induced by the advent of paraconsistent logics, these logics have known an ever greater popularity and turned out to have many applications. Today they are generally recognized as closure operations that may be unpopular with some scholars and that are clearly weaker than some (though not all) other closure operations, but that are obviously sensible. I do not believe that paraconsistent logics, let alone a single one, will in the future be seen as the standard of reasoning, or even as the standard of deductive reasoning. Yet I am convinced that a large number of paraconsistent theories will originate in the near future and will become popular in view of their interesting properties —I shall occasionally give examples in subsequent sections. So I expect that paraconsistent logics will play a much greater role in the future. Yet, their advent and expected popularity does not, in my view, offer any new arguments for logical pluralism or any new challenges to it.

This seems the best point to insert two warnings. The first is that paraconsistency and pluralism are both fascinating and complex and that they concern basic views on logic and on knowledge. Such views are often implicit in the writings as well as in the thinking of scholars. So misunderstanding is likely and the reader better treads carefully, especially if he or she considers some claims as obviously mistaken or even nonsensical.

The second warning concerns conventions. Unless specified differently, I shall use the word "logic" in the broad sense. It will denote any function $\mathbf{L}$: $\wp(\mathcal{W}) \to \wp(\mathcal{W})$, also noted as $\Gamma \mapsto Cn_{\mathbf{L}}(\Gamma)$—so a function that assigns a consequence set $Cn_{\mathbf{L}}(\Gamma)$ to every premise set Γ. The second convention is that (i) the metalanguage in this paper will always be classical and (ii) "*A* is false" will function as classical negation of "*A* is true". This may seem dubious, but I shall argue later in this paper that it is possible as well as sensible to follow the convention.

2. Epistemic Pluralism

The traditional Western ideal of knowledge sees knowledge as a unified and monolithic body, which is justified by a mechanism that may be external or internal with respect to the body of knowledge. There are several views on this body. I shall not spell them out but rather argue that the traditional Western ideal is mistaken in several respects.

(1) The body of knowledge forms a patchwork rather than a unity. It comprises a large variety of domains. There are the 'sciences': mathematics, physics, chemistry, biology, psychology, sociology, economy… and these are split up in disciplines and subdisciplines. The so-called theories of these subdisciplines are partial and incomplete in the sense that more knowledge is hoped for in the future. Of these theories only a few are axiomatized and those that are, are affected by the limitative theorems; they are not provably consistent and they are incomplete in several senses.

Apart from the odd qualities of the 'theories', they are only in part amalgamated.

Many couples of disciplines and subdisciplines are simply unrelated.

(2) The situation is heavily complicated by the fact that alternatives are available for many theories. In mathematics the examples are well-known: set theories, geometries, and so on. Sometimes an alternative is simply an extension of an older theory. Sometimes the alternatives are incompatible with each other. Sometimes it is unknown whether they are compatible. Empirical theories are nearly always competing with each other. [1] String theories vs. elementary particle theories form a ready example. Yet the example is misleading in that it concerns a well-known and striking conflict in a fundamental discipline. Whenever one looks closer into any discipline or subdiscipline, variants are readily located. In the social sciences they are overwhelming.

(3) A unique *concrete* framework for the whole body of knowledge is absent.

As a result, a standard procedure for extending our knowledge in such a way that some parts are better connected is absent. For example, even materialistic and reductionistic psychologists have no clear idea of the

[1] As no (actual) empirical theories are axiomatized, variants to theories are only recognized if striking and fundamental differences occur.

physical explanation of human behaviour and do not know an obvious way that would lead to such an explanation.

This should not be misunderstood. Pursuing unification is extremely important. It is an essential motor of the 'internal' dynamics of disciplines. The pursuit raises *specific problems* for certain theories. One may, for example, attempt to adjust cognitive psychology to specific insights from brain physiology, or one may try to develop a mathematical tool for the benefit of an empirical discipline. Such steps are extremely important. However, in the absence of a unifying framework, no standard procedures are available and the unifying steps will require a creative problem solving process.[2]

So unification is important and a unified body of knowledge is a sensible ideal. Yet, one cannot understand the present pluralism in the sciences and the need for logical pluralism by concentrating on that ideal and neglecting the present state of knowledge.

(4) The situation is even more dramatic in view of the central role of methods and cognitive values. Descriptive knowledge theories need to be justified, rejected, transformed, and sometimes selected among competitors. The required methods and values are not God given.[3] Quite to the contrary. Methods are delineated by methodological theories and these are subject to transformation. The transformations may be caused by reasoning—so ultimately by an attempt to unification—but are more often originated by empirical data or other changes to descriptive knowledge. Cognitive values are equally subject to transformation. Moreover, as for other values, their application to novel or somewhat complex cases requires interpretation, specification, and sometimes even modification.

The justification of methods and cognitive values, or of theories about them, involves many hard problems. I present only a few examples without much discussion. There are many philosophical problems, the most popular one being the so-called fact-norm gap. Central in this is the role of

[2] By this I mean a process that necessarily will consist of a sequence of problem solving situations (or contexts) as well as a variety of 'derived' problems, at least some of which result from analysing the reasons for not obtaining an answer to some of the involved questions. The more general picture is sketched in several publications (Batens 1985, 2001, 2003, 2004, 2007, 2014a; Meheus 1995, 1999a, 1999b, 2000a).

[3] They are not given by our intuitions either. Actual intuitions are acquired by experience, and hence unable to provide a watertight warrant. The belief in warranting intuitions depends on the pre-critical idea that a God would have implanted them in us.

descriptive knowledge in the justification of methods. One may try to explicate this role by an end-means reasoning. However, this leads us to even more fundamental philosophical disagreements. Thus the goal of acquiring knowledge may be related to truth –it is by the traditional definition of knowledge. Some will prefer a pragmatic justification; others a justification in terms of problem solving, as was popular in the second half of last century, most explicitly in Larry Laudan's approach (Laudan 1977).[4] Next to the philosophical problems there are computational problems that surface in certain applications of the methods and values. The reason for this is that the reasoning necessarily proceeds in predicative terms, but that it is moreover always defeasible. I offered examples elsewhere (Batens 2004), showing that the results of applications of the methods do not form a semi-recursive set.

What is the impact of all this? Changes to one theory often cause changes to others. Obviously changes to theories on methods and cognitive values affect the justification of descriptive theories. And the opposite effect also occurs.

Methodological theories have factual presuppositions that may be falsified or may become extremely unlikely.

(5) Descriptive theories, methodological theories, conceptual systems or languages, and logics are modified and replaced. Change in one of the elements often causes change in others. No one with any knowledge of the history of the sciences will have any doubt about this claim, except perhaps where it pertains to logics. Even the changes to logics are clear enough for the non-prejudged observer, but those logicians who believe in the existence of a unique 'true logic', will try to reason them away. Of course, this does not entail that they are mistaken. So let us consider the claim unsubstantiated in as far as logics are concerned. There is more to come on this.

(6) The changes mentioned in (5) are largely unpredictable. It is unpredictable which problems will occur before they do occur. More importantly, before those problems occur, and often a good while thereafter, it is impossible to delineate the set from which the modified theories, methods, and so on, will be chosen.

The point is especially consequential where it concerns conceptual changes. As long as the problem that causes the change does not occur, scientists usually cannot even imagine the conceptual situation that will

[4] A nice illustration of the discussions that might result was also presented by Laudan (1990).

result from the change, let alone the effect of this situation on descriptive theories, methods and values, languages and logics. So the situation which is taken to be actual after the change, could not by anyone have been seen as a possibility a while earlier.[5] The implications for claims on *logical possibility* and related topics is evident. If logical possibilities merely concern the meanings of logical symbols, as is the case in the semantics of a logic or in model theory, they are fully irrelevant for the possible state of the world as expressed by denoting terms. If logical possibilities also pertain to the meaning of those terms, they are language dependent and hence fail to involve future conceptual changes. It is almost embarrassing to say so. Whoever stated anything different after Carnap's work on state descriptions (Carnap 1947) should at least have adduced some good arguments.

It is usually not difficult to incorporate older conceptual systems into newer ones. The old views thus become logical possibilities. In general, it is often easy enough to forge different conceptual systems, even if they contradict each other, into a single one. They contradict each other because their presuppositions contradict each other and the presuppositions usually boil down to existence claims concerning entities (objects, facts, processes...), sets of entities, sets of tuples of entities, etc. If two conceptual systems do not differ too much, for example in that they require the same entities, it is sufficient to widen the possibilities in such a way that the presuppositions become mere possibilities. So the limits of a logical space depend on the entities it is defined over and on its unseen presuppositions.

(7) The changes to theories, methods, languages and logics require reasoning about those entities. So it may be said that the reasoning proceeds at a higher level.[6] A level is defined by (at least) a language (and conceptual system), a logic and a set of methods—methods may be seen as defeasible consequence relations that extend premise sets.

Claim (7) does not require that there is a highest level which is stable or that a higher level does or does not share its logic with lower levels, or that higher levels are justified (or even defined) independently of lower levels. Obviously, there is no highest level. Indeed, one may ask questions about any level: about its concepts, its logic, e.g. inference rules, the correctness of a specific reasoning sequence, and so on. Moreover, given any non-standard

[5] Nice relevant work was produced by Nicholas Rescher (Rescher 2003, 2005; Batens 2008).

[6] There are all kinds of objections against hierarchies of levels, but I do not think that they apply to the existence of such levels as introduced in the text.

problem, no one can predict the highest level that one will have to reach in order to solve the problem.

Where, at the beginning of this section, I denied the existence of a unified and monolithic body of knowledge, the claim was definitely vague and ambiguous. I hope to have repaired this by the specifications introduced within this section.

An unannounced conclusion is the absence of a stable or final highest level and the fact that it is unpredictable to which 'height' a problem solving process will (have to) move before reaching a solution. Levels will turn up again in the next section.

3. Logical Pluralism

Whether logical pluralism is required or not is determined by the function of logic with respect to theories. Three functions have to be distinguished: a logic may function as the underlying logic of a theory, a logic may be invoked to explicate the reasoning that goes on in applications of theories, and a logic may be invoked to explicate the reasoning that leads to transferring statements between theories–which consequences of one theory are transmitted to the other in order to strengthen the latter theory's consequences.

These three functions seem to cover the whole domain, especially if the "theories" are not seen as comprising all knowledge within a certain domain,[7] but as chunks of knowledge, where "knowledge" is loosely defined to cover methods and values. The chunks need some internal organization, they need to be applied, separately or more often in combination. The description of a method or of cognitive values may be seen as a theory, organized by a logic.

Logicians like to define theories as the result of applying a closure operation on a set of non-logical axioms. The set of theorems of the theory is then $Cn_{\mathrm{L}}(\Gamma) = \{A \mid \Gamma \vdash_{\mathrm{L}} A\}$, the set of consequences derivable by the underlying logic L from the set of non-logical axioms Γ.

As we all know, most theories, whether mathematical or empirical, are presented in such a way that the reasoning and proofs are kept 'informal', which here means non-formalized. Traditionally, most logicians seem to have believed that 'logic', in more recent periods specified as classical logic, **CL**, provides the correct explication for the informal reasoning. Yet, there is a set of well-studied

[7] In view of (2) from Section 2, this is often impossible anyway.

mathematical theories that are explicitly meant to have intuitionistic logic, **IL**, or a similar system as its underlying logic. Other logics often serve a rather 'theoretical' function in this respect in that they have no popular applications. This holds, for example, for relevant, paraconsistent, adaptive, and other logics. Where they are the underlying logics of actual theories, the theories are formulated by logicians, usually to make a philosophical or technical point, but are not applied by (a significant number of) scholars in the domains to which the theories pertain.

The second function of logics was to explicate the reasoning involved in applications of theories. Where the domain is non-mathematical, scholars hardly seem to care, leaving the explication to the logicians. Again, the majority of the latter seem to believe that **CL** offers the right explication, but hardly ever attempt to actually offer an explication. They apparently do so because they believe that **CL** is the 'true logic' anyway. This is not very convincing, especially as hardly any reserve is shown in cases where **CL** obviously runs into trouble, as is for example the case for Quantum Mechanics.

The third and final function was to explicate the reasoning that regulates the transfer between theories, or also between knowledge chunks. In specific cases, one theory is seen as extending the other, for example when mechanics is considered to incorporate analysis. This may be considered as sensible, but it is still odd that mathematical theorems would be seen as theorems of mechanics. So it seems more natural to opt for an inferential approach: inferences justified by analysis allow one to derive theorems of mechanics from other such theorems (or axioms, where the term makes sense). Still, the construction is not obvious and if the deductive logic is merely **CL**, the oddities one tries to avoid will obviously be unavoidable.

Where the theories or knowledge chunks contradict each other, as is the case for counterfactual reasoning, such an inferential approach is not only odd but trivializing. Special systems have been developed, which I prefer to label procedural—procedures are introduced below. Examples of such systems were developed by Nicholas Rescher, partly with Ruth Manor (Rescher 1964, 1970), by Peter Schotch and Raymond Jennings (1989), by Bryson Brown and Graham Priest (2004). Many other procedures are obviously possible.[8]

The provisional conclusion is that the choice of a logic should mainly be assessed in view of the explication it provides. Stressing the notion of explication

[8] The aim of adaptive logics, introduced below, is to characterize all such procedures, and many more, by dynamic proof procedures and a selection semantics, possibly under a translation.

is essential. What is 'out there' is actual reasoning, *not* a domain that can be described, *neither* a matter of fact *nor* a platonic heaven. Indeed, the explicandum unavoidably contains mistakes and there is a normative dimension. We are not interested in describing the reasoning as it occurred, but in its justifiable reconstruction. This holds for all three functions of logic.

With all this in mind, let us turn to the traditional classification of logic. First there is deductive reasoning, which is determined by the meaning of terms; formal deductive reasoning is determined by the meanings of the logical terms, informal deductive reasoning by the meanings of the non-logical terms (or referring terms). Next there is defeasible reasoning, which basically comes to 'methodological' reasoning. The correctness of defeasible reasoning is not determined by the meanings of any linguistic entities, although it obviously is influenced by them. The correctness of defeasible reasoning is determined by the criteria that govern the justification of methods: whether their results serves their purpose, the quality of those results, the efficiency of the process that leads to them, and so on.

So let us, finally, consider logical pluralism in terms of this classification. Even in the domain of formal deductive reasoning, an overwhelming amount of arguments supports the need for logical pluralism. For one thing, there is a plurality of logical terms that go beyond the scope of the common systems called logics and that occur in specific linguistic contexts. Examples are the sundry kinds of modalities—alethic, deontic, pragmatic, etc.—of causal relations, of time and tense operators, and so on. Moreover, none of these logical terms seems to be unique. Does anyone even have the beginning of an argument for the thesis that only one negation, one implication, one conjunction... occurs in actual reasoning? And there are a couple of serious counterarguments. One of them is that all those monologists adduce at best theoretical arguments, but never an empirical study of actual reasoning or even of actual texts. That, moreover, monologists quarrel among themselves about the logic that is the one and only—classical, intuitionist, relevant, etc.—is not really an objection, but neither does it support their so-called self-evident position.

But apparently monologism is being softened. More recent monologist brands actually advertise logics in which a plurality of logical symbols occurs. Typically most relevant logicians admit that there are several sensible implications and that some of them serve specific purposes better than others. Thus Anderson, Belnap and Dunn (1992) admit that **R** rather than **E** is required for empirical theories and many other relevant logicians argued for picking a specific relevant implication for a specific purpose (Routley 1982, Brady 2006, Weber 2010). And once there is a manifold of implications, there is a manifold of definable negations (like $A \rightarrow \perp$), disjunctions (like $*A \rightarrow B$ for a unary $*$) and conjunctions (like $*(A \rightarrow *B)$ for a unary $*$). Of course this puts logical monists in a situation that closely resembles the one of pluralists which monists often find objectionable: that one needs criteria to determine which (sequences of) words from natural language should be explicated by which (sequences of) formal symbols.

If the one and only logic has a manifold of unambiguous logical terms, there is obviously no reason why all of them should occur in all knowledge chunks (or theories) or in all reasoning that concerns the application of knowledge chunks or the transfer between them. So the explication of reasoning in different contexts proceeds in terms of sublogics of the one and only logic. But why is that simpler or better in any other way than the situation in which mutually incompatible logics do the job in the different contexts? By mutually incompatible logics I mean logics that cannot be forged into sublogics of the same logic.

Next, consider a situation in which logic has to serve one of its three functions. Why should a unique logic offer a suitable explication of the involved logical terms? Clearly several logics may be equally good in this respect, either because they lead to the same results, viz. the same consequence set, or because none of the consequence sets is arguably superior. The argument would seem inescapable even if we were dealing with a description, and it is the more convincing as we are dealing with an explication. I cannot see in which way a monist might rebut the argument, unless by starting from the presupposition that all correct reasoning is the monopoly of a single logic—call this the monopoly presupposition.

Suppose that a logic L provides a suitable explication of the logical terms that occur in a context in which we reason about the beauty of Bach's Cello Suites. Why then should L be a suitable explication of the logical terms that occur in the context in which we reason about **L**? Clearly the non-logical terms of both contexts are very different. So why, if one does not rely on the monopoly presupposition, should the logical terms be identical? So why then should the metatheory of a logic **L** have **L** itself as its logic?[9]

If different contexts require different logics, there is no foolproof trick to

[9] It is interesting to check whether a logic can function as the underlying logic of its own metatheory and, if so, how adequate or complete the resulting metatheory is. But is this more than a technical feature of the logic?

build an embracing logic that equally well explicates the reasoning in the different contexts. The point is that logics define consequence relations. For this reason, there is no warrant that the embracing logic is a conservative extension of the embraced logics. Put differently, joining the logics might change the meaning of some of the logical terms.

Some may try to save the idea of a one and only logic by seeing it as a so-called umbrella logic. Given a set *S* of logics, the umbrella logic would determine which member of *S* is adequate in which context. This way out does not save logical monism. The umbrella logic is clearly not a logic in the usual sense, but rather an instruction or a method for choosing logics. It does not even assign a unique consequence set to every premise set, among other things because several members of *S* may be equally suitable for certain contexts.

All these, it seems to me, are good arguments against different brands of logical monism and jointly form a good reason to consider pluralism as the only viable alternative. As far as non-formal reasoning is concerned, monism does not even seem sensible. Nor was it defended by anyone as far as I know. Apart from the fact that most arguments concerning formal reasoning carry over here, referring terms are vague and ambiguous, and moreover theory-laden.[10] Most of what was said about deductive reasoning also carries over to defeasible reasoning, as do the insights on epistemic pluralism. More importantly, the idea that methods would be a priori was replaced by the view that they are learned in scientific practice, in which we learn how to learn (Shapere 2004). An oddity that may be added at this point is that defeasible logics may function as underlying logics of complex theories.[11]

The present discussion would be incomplete if I did not consider the question: How to justify the choice between logics for a specific explication? I already hinted towards some answers, but let me recapitulate explicitly. Let me begin by admitting that lots of work still has to be done in studying properties of logics that determine their suitability for a specific situation. Yet, as we start from reasoning to be explicated, a few general guidelines are obvious. Clearly the explication should not trivialize the context. Next, it should recapture the bulk of the given reasoning. There is no need for trying to recapture 'as much as

[10] This does not result in incommensurability and does not exclude communication.

[11] In the case of adaptive logics, for example, theories may be up to Π^1_1—complex (Batens, De Clercq, Verdée, and Meheus, 2009; Verdée 2004). Several examples have been published and more are underway (Verdée 2013a, 2013b; Batens 2014b, in preparation).

possible' and definitely no need for determining the logic that maximizes the consequence set. The task is a matter of satisficing rather than optimizing. In practice one should pick a choice and look for counterarguments; a 'pragmatic' and provisional choice will do.[12]

4. When Inconsistency Emerges

Consider a theory $T = \langle \Gamma, \mathbf{CL} \rangle$ that is meant as and believed to be consistent and hence is given **CL** as its underlying logic. Suppose that T turns out inconsistent. So $Cn_{\mathbf{CL}}(\Gamma) = \mathcal{W}$ whence T is trivial. As a result, one will try to devise a consistent replacement $T' = \langle \Gamma', \mathbf{CL} \rangle$ for T. This does not mean that T is abandoned and T' is built from scratch. Quite to the contrary, (P1) one will reason from T in order to locate the inconsistencies in it and in order to obtain a maximally consistent 'interpretation' of T,[13] and (P2) one will try to devise one or several consistent replacements T' from there—the two steps are given a name for future reference.

Solving problem (P1) requires a systematic and formal method, whereas the choices involved in (P2) are not justifiable on logical grounds but depend on specific properties of Γ and possibly on new information.[14] It is the aim of this section to review a few candidates for (P1).

A blunt approach consists in *going paraconsistent*. This means that, in $T = \langle \Gamma, \mathbf{CL} \rangle$, one replaces **CL** by a paraconsistent logic **P** which is defined over the same language as **CL**. A logic P is paraconsistent iff $A, \neg A \vdash_{\mathbf{P}} B$ is invalid. So **P** invalidates certain **CL**-inferences. Actually, nearly all paraconsistent logics invalidate Disjunctive Syllogism (from $A \vee B$ and $\neg A$ to derive B).[15]

If **P** is a *Tarski* logic,[16] then $Cn_{\mathbf{P}}(\Gamma)$ is too poor. This may be seen by considering a simple propositional example.

$\Gamma_1 = \{p, q, \neg p \vee r, \neg q \vee s, \neg q\}$

[12] There is a huge number of inconsistency-adaptive logics. Over the years, quite a few rules of thumb were acquired for choosing one of those logics for a specific purpose. It would be too odd a digression to discuss those rules of thumb here.

[13] The quotation marks warn the reader that an informal interpretation is meant, not an interpretation as provided by a semantics. The informal interpretation delineates a certain set of statements that the premises affirm.

[14] Actually, the matter is slightly more complicated in that several approaches may lead to "a maximally consistent 'interpretation' of T"; see the passage on "other corrective adaptive logics" below in the text.

[15] Disjunctive Syllogism together with Addition (to derive $A \vee B$ from A) warrant *Ex Falso Quodlibet*, also called Explosion (to derive B from A and $\neg A$) if the consequence relation is Transitive.

[16] A logic **L** is Tarski iff it is reflexive ($\Gamma \subseteq Cn_{\mathbf{L}}(\Gamma)$), monotonic ($Cn_{\mathbf{L}}(\Gamma) \subseteq Cn_{\mathbf{L}}(\Gamma \cup \Delta)$ for all Δ) and transitive (if $\Delta \subseteq Cn_{\mathbf{L}}(\Gamma)$ then $Cn_{\mathbf{L}}(\Gamma) \subseteq Cn_{\mathbf{L}}(\Gamma)$).

As Γ_1 is (explicitly) inconsistent, $Cn_{\mathbf{CL}}(\Gamma_1)$ is trivial. A maximally consistent 'interpretation' of Γ_1 warrants that r is a consequence of it. Indeed, if all propositional letters are consistent whenever Γ_1 does not require them to be inconsistent, then p is true and $\neg p$ false, and so the truth of $\neg p \vee r$ guarantees that r is true.

It is easily seen that no Tarski logic offers an adequate approach. Let L be a paraconsistent Tarski logic. Case 1: **L** validates Addition. So Disjunctive Syllogism is invalid in **L** as explained in footnote 15. Case 2: Addition is invalid in **L**. Then **L** does not offer a maximally consistent interpretation of Γ_1 because infinitely many members of $Cn_{\mathbf{CL}}(\Gamma_1)$ are absent from $Cn_{\mathbf{L}}(\Gamma_1)$ but are verified by the minimally abnormal **L**-models of Γ_1, for example the formula $p \vee A$ for all A.[17]

A different approach proceeds in terms of a *procedure*. This is a set of *instructions* that lead from a premise set to a construction, which is a proof-like entity. An instruction should be seen as a couple consisting of a rule and a permission or obligation that depends on (i) the premise set and (ii) the state of the construction. The form of this permission or obligation is something like: if A is a premise and $X_1, \ldots, X_n$ occur in the construction and $Y_1, \ldots, Y_m$ do not, then one may/should add Z to the construction—the X, Y and Z are certain entities from which the construction is built.[18]

Procedures were introduced, for example by Paul Weingartner (2000). They have certain advantages over logics in that they allow one, for example, to define goal directed **CL**-proofs (Batens and Provijn 2000). A weaker procedure defines the logic (in the broad sense) $\mathbf{CL}^{-}$ (2012). The latter is just like the procedure for **CL** except that Ex Falso Quodlibet is invalid.

No procedure will be spelled out in the present paper, but the main idea may be clarified as follows. A $\mathbf{CL}^{-}$-consequence is a formula that can be *constructed* from the premises by the procedure. For this reason $\mathbf{CL}^{-}$ is able to validate Addition as well as Disjunctive Syllogism without validating Ex Falso Quodlibet. Let $\Gamma_2 = \{p, \neg p\}$. There is a construction for $\Gamma_2 \vdash p \vee q$: as the goal is $p \vee q$, look for a premise of which $p \vee q$ is a positive part—a notion defined by Schütte (1960). There is no such premise but the procedure allows one to transform the 'goal set' $[p \vee q]$ to two goal sets, $[p]$ and $[q]$. These allow one to introduce a premise of

[17] This is the situation for the (somewhat special) logic AN (Meheus 2000b). As expected, the adaptive version, called ANA, validates all those formulas.

[18] If instructions are taken as primitive, the rules by which usual logic proofs are built may be defined as instructions combined with a universal permission. Both procedures and the usual logics are logics in the broad sense as defined in Section 1.

which p, respectively q, is a positive part. As p itself is a premise, the $\mathbf{CL}^{-}$-construction succeeds, whence $\Gamma_2 \vdash_{\mathrm{CL}^-} p \vee q$. By a similar construction, $p \vee q, \neg p \vdash_{\mathrm{CL}^-} q$ because the premise $p \vee q$ allows one to obtain q if one obtains $\neg p$ and that is itself a premise. But there is no way to obtain $\Gamma_2 \vdash_{\mathrm{CL}^-} q$: the goal set $[q]$ cannot be further analysed and there is no premise of which q is a positive part.

Let us move on to the next approach: *inconsistency-adaptive logics* (Batens 2015). There is a paraconsistent logic **P** that functions as the so-called *lower limit*: whatever is derivable by **P** is unconditionally derivable. The central adaptive idea is that abnormalities[19] are considered as false unless and until shown otherwise.

Lines of the annotated dynamic proofs are like lines of usual annotated proofs, but there is a further element on each line, viz. a condition. Premises are introduced on the empty condition. The rules of inferences of the lower limit are unconditionally valid. Next, the gain provided by the adaptive logic may be characterized, at the base level, by the following central deductive mechanism.

$$\frac{A \vee \exists(B \wedge \neg B) \qquad\qquad \Delta}{A \qquad\qquad \Delta \cup \{\exists(B \wedge \neg B)\}}.$$

So when the disjunction of a formula A and an abnormality $\exists(B \wedge \neg B)$ is derived on a condition Δ, the abnormality may be pushed to the condition. The meaning of this is that the formula A is derivable provided the premises allow one to consider the abnormality as well as the other members of the condition as false. Also central to the dynamic proofs is the marking definition: lines are *marked* in view of the minimal disjunctions of abnormalities that are *unconditionally* derivable—derivable on the condition $\emptyset$—from the premises. If a line is marked, its formula is taken not to be derivable by the insights provided by the stage of the proof—the proof moves to the next stage whenever a line is added.[20] Next to this *derivability at a stage*, there is a notion of *final derivability*, which is defined in terms of possible extensions of proofs. That final derivability is not decidable at the predicative level is the reason why the proofs are dynamic in the first place. The semantics of adaptive logics is a selection semantics.

[19] The abnormalities of inconsistency-adaptive logics are inconsistencies, or rather specific forms of them. In the present section, I shall only consider abnormalities of the form $\exists(A \wedge \neg A)$, in which $\exists$ abbreviates an existential quantifier over every variable free in A.

[20] So a proof stage may be seen as a sequence of lines and a dynamic proof as a chain of stages.

Models of the premise set are selected in view of the set of abnormalities they verify; the adaptive semantic consequences are the formulas verified by all selected models.

It is helpful to specify the following. A paraconsistent logic **P** may be related to **CL** by saying that **CL** validates certain rules on top of those validated by **P**. Inconsistency-adaptive logics validate certain *applications* of such rules– which applications are validated depends on the disjunctions of abnormalities that are derivable from the premise set by the lower limit logic. Looking at the same picture from the opposite point of view, replacing **CL** by a paraconsistent logic comes to considering certain **CL**-rules as invalid; [21] replacing **CL** by an inconsistency-adaptive logic comes to the same, except that *applications* of the invalidated rules are retained whenever the involved formulas can be taken to be consistent.[22]

The strengths of inconsistency-adaptive logics are threefold. (i) Their dynamic proof theory explicates defeasible reasoning–the point is substantiated in many papers (Batens 2015). (ii) Inconsistency-adaptive logics define consequence sets that are much less dependent on the formulation than for example $\mathbf{CL}^-$-consequence sets. (iii) There is a large variety of inconsistency-adaptive logics, the variation pertaining to the lower limit logic, the abnormalities, the strategies, and the way in which inconsistency-adaptive logics are combined with each other.

Problem (P1) does not require deductive inferences but *methodological decisions*, and that is precisely what adaptive logics are able to explicate. This also clarifies why it is an advantage that there is a multiplicity of inconsistency-adaptive logics: the choices to be made do not concern the meanings of logical symbols but are methodological in nature.

Comparing inconsistency-adaptive logics with the procedure $\mathbf{CL}^-$ provides useful insight. Where Γ is an inconsistent premise set, $\mathbf{CL}^-$ avoids triviality because the procedure analyses the goal into 'targets', and analyses premises in view of those targets, but does not allow one to derive formulas that are not obtained as targets.[23] The behaviour of an inconsistency-adaptive logic depends on the set of formulas that can be taken to behave consistently. This, in turn, is

[21] For pragmatic reasons **CL** is chosen as the *upper limit* logic.

[22] This is the intuitive idea. Inconsistency-adaptive logics originated by making the idea precise and systematic.

[23] Referring to a previous example, q is not derivable from $\{p, \neg p\}$ because q is the only target and is not a positive part of one of the premises.

determined by the lower limit consequence set of the premises.

A further way to handle inconsistent premise sets are (what I like to call) *Rescher-Manor mechanisms* (Rescher 1964; Rescher and Manor 1970); many variants, both 'flat' and 'prioritized' were meanwhile studied (Benferhat, Dubois, and Prade 1970, 1999). Rescher-Manor mechanisms proceed in terms of maximal consistent subsets (m.c.s.) of the premise set. The m.c.s. of, for example, Γ_1 are $\{p, \neg p \vee r, \neg q \vee s, \neg q\}$ and $\{p, q, \neg p \vee r, \neg q \vee s\}$. A formula is a Weak consequence of Γ iff it is a **CL**-consequence of a m.c.s. of Γ; it is a Strong consequence of Γ iff it is a **CL**-consequence of all m.c.s. of Γ; and there are more kinds of consequences. Note that Rescher-Manor mechanisms are heavily dependent on the formulation of the premises. Thus both p and r are among the Strong consequences of Γ_1, but neither is a Strong consequence of $\{p \wedge q, \neg p \vee r, \neg q \vee s, \neg q\}$.

Rescher-Manor mechanisms are characterized by an inconsistency-adaptive logic under a translation (Batens 2000; Verhoeven 2001, 2003). This is useful. At the predicative level Rescher-Manor mechanisms are computationally hopeless; the set of consistent subsets of a predicative premise set is not in general semi-recursive. The adaptive characterization provides the mechanisms with a dynamic proof theory. On the one hand, this defines the complex consequence set; on the other hand it explicates our reasoning towards it.

There are other means to handle the problem (P1) and I only mention one of my preferred ways: *other corrective adaptive logics*–these are adaptive logics that assign to certain premise sets a consequence set that is non-trivial and moreover 'interprets' the premises 'as consistently as possible'. [24] The 'interpretation' should be obtained on formal logical grounds. Thus, if the premises affirm $(p \wedge \neg p) \vee (q \wedge \neg q)$ and no other relevant stuff about p and q, then the two disjuncts should be treated on a par.

It turns out (Batens 2016) that, in order to obtain a maximally consistent 'interpretation' of a premise set, it is not always required to proceed in paraconsistent terms. That $\neg A$ is true in case A is also true may be seen as a negation glut– **CL**-models that verify A falsify $\neg A$, whereas here $\neg A$ is true. Similarly, that $\neg A$ is false in case A is also false may be seen as a negation gap. If both A and B are true, and $A \wedge B$ is false, we have a conjunction gap; a conjunction glut obtains if either A or B is false, and $A \wedge B$ is true. And so on for all other logical symbols.

[24] The quotation marks indicate that the phrase is not unambiguous. The phrase is disambiguated by the adaptive strategy, each strategy defining a sensible way to interpret the phrase.

Consider, by way of example,

$\Gamma_3 = \{p, r, \neg q \vee \neg r, (p \wedge r) \supset q, \neg p \vee s\}$.

Γ_3 has obviously paraconsistent models—models of logics that allow for negation gluts. However, it has also models of logics that allow for conjunction gaps and it has models of logics that allow for disjunction gluts. So allowing for other abnormalities than negation gluts and minimizing those abnormalities results, for some premise sets, in minimally abnormal 'interpretations'. These form a sensible starting point for solving (P2), a starting point just as sensible as a minimally inconsistent 'interpretation' of Γ_3.[25]

Some corrective adaptive logics do not minimize gluts and gaps with respect to logical symbols, but minimize ambiguities with respect to non-logical symbols—I skip the technical trick to realize this. Next, all such corrective adaptive logics, those that minimize gluts and gaps as well as those that minimize abnormalities may be combined—some combinations combine two of them, or three of them, . . . , up to all of them. The lower limit logic is called zero logic, **CL∅**, because nothing is a consequence of anything else—even $p \nvdash_{\mathbf{CL}\emptyset} p$. However, adaptive zero logic, for example **CL∅**^{m}, assigns a sensible minimally abnormal 'interpretation' to every premise set.

Two more comments and then I move on to the next topic. First, none of the preceding can be seen as an application of **CL** itself. Even when the consequence relations are defined in terms of **CL**, as is the case for the Rescher-Manor mechanisms, they are obviously different from **CL** because they assign non-trivial consequence sets to at least some inconsistent premise sets. The second comment is that, with the exception of paraconsistent Tarski logics, all the approaches assign to consistent premise sets exactly the same consequence set as **CL** assigns to them. This is a further nice property which they all share, apart from being paraconsistent. This also shows that, unlike what many classical logicians and people like Peter Vickers (2013) seem to think, one cannot see from texts that their authors apply **CL**. All one can see is that they apply either **CL** or one of those logics in the broad sense that assign to consistent premise sets the same consequence set as **CL**. This is especially relevant when commenting on scholars that did not know **CL**, which includes everyone who lived before 1900 but also most scientists living thereafter.

[25] Every premise set that requires a glut or gap or ambiguity to obtain—see below in the text—is inconsistent. But that does not mean that the paraconsistent road is the only one or even the best one.

5. Some Theses

In this section I try to summarize my position with respect to the questions in the call for papers for this volume. Clarity and structure seem best served by phrasing some theses with comments.

T1 Paraconsistent logics are required (i) where inconsistency emerges within a theory or domain or between theories, (ii) for handling counterfactuals and related notions (laws, causality...), and (iii) for handling interesting inconsistent theories, whether empirical or mathematical.

Arguments for (i) are found in the history of the sciences. At this point, I have again to warn the reader for a mistaken view, defended among others by Peter Vickers in his otherwise very interesting book (2013). The view is that scientists do not draw any consequences from inconsistencies, and hence that the presence of Ex Falso Quodlibet in **CL** is harmless. The trouble is especially with the second half. It stems from the mistaken claim that no consequences of an inconsistency can be derived by **CL** as long as the inconsistency itself was not derived. This is obviously wrong and naive. When an inconsistency is discovered within a theory, one cannot consider the *other* statements that were derived by **CL** from the theory as safe. The fact that predicative **CL**-consequence sets are not semi-recursive and that there is no positive test for consistency—that the set of consistent sets is not semi-recursive—turns the objection into a fatal one.

While, in T1, (ii) is a simple fact,[26] (iii) might be debatable in view of disagreements on what is interesting. It seems to me, for example, that it is interesting that some inconsistent set theories are provably non-trivial, especially as none of the 'classical' set theories is provably non-trivial (Brady 2006; Verdée 2013a, 2013b; Batens, in preparation).

Let us turn to consequences of T1 for logical pluralism. As was shown in Section 4, there is a multiplicity of minimally abnormal interpretations of theories. Several of these are viable in most contexts, viz. when dealing with (P1) in connection with a specific inconsistent theory. It is correct that there are some insights on the suitability of specific approaches and we should try to increase these. One knows, for example that some lower limit logics are more suitable

[26] Some definitions proceed in terms of modal logics, but the criteriology does not.

than others for specific purposes. Empirical and mathematical theories impose different demands in this respect. Similar insights allow one to rule out certain adaptive logics for a given purpose. Moreover, the careful study of the theory under consideration will often reveal specific demands and show certain adaptive approaches inadequate—there is no foolproof method that adequately solves (P1) for all situations. Still, a manifold of (adaptive and other) approaches will in each specific case make sense and, more importantly, be the correct road to a solution of (P1) that allows for an interesting solution to (P2).

The claims in the previous paragraph concern logics in the broad sense, means to characterize methods. That there is a manifold of methods may not come as astonishing. The situation is different for (ii) and (iii). My reference section is terribly incomplete with respect to (iii), the number of underlying logics is enormous and I do not see any way to rephrase all of the theories, or even theories close to the available ones, in terms of a single underlying logic.

T2 It is unsettled whether our *best* future theories will be consistent or inconsistent.

One of the arguments, which I take as convincing, is that the complexity of the world may prevent a consistent description in a denumerable language. As humans are taken to be unable to handle non-denumerable languages, T2 follows. I must add, but I said so elsewhere, that I am unable to understand consistent or inconsistent unless as a property of linguistic entities. So I fail to understand Dialetheism as standardly defined. Of course, one may understand it as stating our best future theories will be inconsistent. I see no argument that justifies ruling this out as an actual possibility—not just a logical one.

What is the relevance for logical pluralism? One might *define* 'the true logic' as the logic that will be suitable for our theories 'at the end of time'. The theories I have in mind are those that our successors will hold on the proverbial day when the dynamics of science will have halted and all scientific problems will have been answered. T2 rules out that one would be able to identify at present 'the true logic' in this sense. Even supposing that some or all of our theories will be inconsistent at the end of time, I see nothing that would at present support the superiority of a specific paraconsistent logic over others. So, in my view, *even if 'the true logic' exists, it is not knowable today.*

Note also that, if it exists, the true logic in that sense is useless for our present purposes and tasks. There is no reason why the logic that would underlie all our theories and their applications at the proverbial stable end of time would have any use for us, who try to move from the present turmoil in the direction of that stable state.

However, the situation is far worse than I depicted it. There is no reason at all to believe that there will be a unique logic at the proverbial end of time. The end state would be more than ideal enough if every theory were stable, if the logics underlying applications was stable, and if the logics for transfer between theories were stable. That all theories would have a different underlying logic, that different kinds of applications would demand for a different logic, that a diversity of logics would be required for the transfer between theories, all this would not in the least diminish the ideal character of the end state.

What matters about a theory is its set of theorems. The way in which the theorems are organized, respectively axiomatized, is far less important. It seems to me that logicians would make themselves more useful by offering simple and transparent means to organize a theory than by advertising their own true logic, often the mere result of prejudice, speculation and lack of imagination.

T3 If an inconsistent theory T is 'translated', without loss of discrimination, into a consistent theory T', then T' allows for more discrimination than T.

I take it that there is no loss of discrimination when logically non-equivalent formulas of T correspond to logically non-equivalent formulas of T'. An example of the intended translation is when the triad of possibilities

Px	$Px \wedge \neg Px$	$\neg Px$

is replaced by the triad

$P_p x \wedge \neg P_n x$	$P_p x \wedge P_n x$	$\neg P_p x$

in which the subscripts refer conventionally to positive and negative. So the inconsistency is removed from the logical space: a single unary predicate, which defines an inconsistent logical space, is translated in terms of two unary predicates.

Such translation is seldom possible. The only simple transformation by which one seems to try to restore consistency is restriction. For set theory, for example, certain entities, like the Russell set, were either removed from the theory or were turned into entities of a special kind, like classes, which have weaker properties, for example cannot be the first element of a member of the membership relation. Other transitions to a consistent theory require more complex transformations to concepts, to empirical criteria, and so on. So actually, there is a methodological pluralism in removing inconsistency.

> T4 Where **L** is a paraconsistent logic and $\mathbf{L}_1$ is defined by the consistent **L**-models, if Γ has $\mathbf{L}_1$-models, then $Cn_{\mathbf{L}1}(\Gamma)$ is non-trivial and, weird cases aside, richer than $Cn_{\mathbf{L}}(\Gamma)$.

An **L**-model is consistent iff, for all formulas A, it falsifies either A or $\neg A$.[27] The $\mathbf{L}_1$-consequences of a premise set are the formulas verified by all consistent **L**-models of the premises.

As Γ has $\mathbf{L}_1$-models, it offers one the choice between two sets of conventions, those of **L** and those of $\mathbf{L}_1$. The latter choice results in a more complete description: as every $\mathbf{L}_1$-model of Γ is a **L**-model of Γ, $Cn_{\mathbf{L}}(\Gamma)\subseteq Cn_{\mathbf{L}1}(\Gamma)$ and, some weird cases aside,[28] $Cn_{\mathbf{L}}(\Gamma)\subset Cn_{\mathbf{L}1}(\Gamma)$.

Under normal conditions, $\mathbf{L}_1$ will validate more rules than **L**. The choice for $\mathbf{L}_1$ will also affect the meaning of the negation: $\neg A$ $\mathbf{L}_1$-entails "if also A, then triviality". Even if this cannot be expressed in the language, we shall have "if $\Gamma \vdash_{\mathrm{L}1} \neg A$, then $\Gamma\cup\{A\}\vdash_{\mathrm{L}1} B$ for all B". In this sense $\neg A$ 'rules out' A, etc. Also, $Cn_{\mathbf{L}1}(\Gamma)$ may be a theory about any topic, for example the description of a logic, viz. of the syntactic and or semantic consequence relation.

As far as logical pluralism is concerned, there is a reason to restrict it in the described situation. Put more correctly, there is a reason to prefer $Cn_{\mathbf{L}1}(\Gamma)$ over $Cn_{\mathbf{L}}(\Gamma)$. But that hardly affects our view on pluralism. Still, dialetheists may balk. They may try to show that Γ, if well understood, is inconsistent anyway. If they can show so, it was a mistake to think that Γ has $\mathbf{L}_1$-models. That is excellent, but is not an objection to T4. Dialetheists may also argue that it is better to stick to **L** because one might obtain new information, which extends Γ and results in an inconsistent Γ'. The situation would for example change if the inconsistent information were likely, but the mere possibility of inconsistent information is not a good reason for staying paraconsistent. Just like the mere possibility of dying is not a good reason for not buying a plane ticket.

[27] The case where **L** is defined over a language with several unary logical symbols is safely left to the reader.
[28] An example of a weird case is where Γ is the set of all formulas verified by a $\mathbf{L}_1$-model.

T5 If some of the best theories are inconsistent, or if the best unified theory is, then it is *almost* straightforward that one can describe the situation in a way that is itself consistent.

Let me first phrase the argument in terms of 'levels'. Let T be a theory. That, for some A, both $\vdash_T A$ and $\vdash_T \neg A$ hold, does not entail that $\vdash_T A$ and $\nvdash_T A$ and does not entail that, for some B, $\vdash_T B$ and $\nvdash_T B$. Of course it is possible that the theory T is defined in an odd way, which causes both $\vdash_T A$ and $\nvdash_T A$ to hold. It is even possible that our usual notion of a theory is deeply defective, and that both $\vdash_T$ A and $\nvdash_T$ A hold as an effect of that. (To be honest, it is possible but unlikely; more unlikely than the inconsistency of Peano Arithmetic for example.) But suppose that both $\vdash_T A$ and $\nvdash_T A$ hold and let us write this as $\vdash_M \vdash_T A$ and $\vdash_M \nvdash_T A$. Why should it follow that $\vdash_M \vdash_T A$ and $\nvdash_M \vdash_T A$ hold? Or why should it follow that, for some B, $\vdash_M \vdash_T B$ and $\nvdash_M \vdash_T B$ hold? And, especially, how would such things follow?

So, as an argument for T5, I would invoke lack of imagination. Why can we not talk about the situation, however inconsistent it may be, in a consistent way? In terms of levels: What would prevent us to move up to a level where we can talk consistently about the inconsistent situation?

I have heard objections against phrasing the argument in terms of levels. The existence of levels would be problematic. Whether this holds water or not, the objection is superficial. Just put the talk at all levels into a single language and rephrase the problem. To illustrate the matter, I need some technical claims. It is easy enough to define a modal logic with the property that negations within the scope of modalities are paraconsistent and negations outside that scope are classical. So $\{\Diamond A, \neg\Diamond A\}$ is a trivial set, but $\{\Diamond A, \Diamond\neg A\}$ is not. Similarly, it is easy enough to define a modal inconsistency-adaptive logic that presupposes consistency unless and until proven otherwise, but does so first for the highest levels. So $\{\Diamond A, \Diamond\neg A\}$ would be preferred to hold over $\{\Diamond A, \neg\Diamond A\}$, the latter would be preferred to hold over $\{\Diamond\Diamond A, \neg\Diamond\Diamond A\}$, etc. So, for any finite premise set, however inconsistent it be, there will be a layer of modalities that is consistent.

Is this foolproof? Of course not, no construction is. And if things go badly wrong, so will dialetheism, whatever its purity.

With respect to logical pluralism, I have argued that a consistent highest level offers a coordinated umbrella view on a plurality of theories and coordinating metatheories, etc., with a diversity of underlying logics.

T3 states that, in certain circumstances, an inconsistent description can be translated into a consistent one. As is stated by T4, there are circumstances in which choosing an explosive logic offers a richer (and consistent) description. Finally, T5 shows that one may consistently describe the patchwork of theories (or chunks) and the plurality of logics that functions within it.

T6 To define higher level inconsistency in terms of lower level inconsistency is objectionable.

It is for example objectionable to stipulate that $\Gamma \nvdash_{L} A$ iff $\Gamma \vdash_{L} \neg A$. Let us have a closer look. Consider a logic like **LP**, in which Excluded Middle holds: in every model *M* either *A* or $\neg A$ has a designated value. In such a semantics, one may safely define $M \nVdash A$ as $M \Vdash \neg A$. Indeed, if *M* assigns a non-designated value to *A*, then it assigns a designated value to $\neg A$; so M $\Vdash \neg A$ and hence $M \nVdash A$ by the definition. If *M* assigns a designated value to both A and $\neg A$, then $M \Vdash A$ will hold together with $M \nVdash A$. Next, define $\Gamma \vDash A$ iff, for all models *M* of Γ, $M \Vdash A$; for a suitable logic of the metalanguage, it follows that $\Gamma \nvDash A$ iff, for a model *M* of Γ, $M \nVdash$A. So $p, \neg p \vDash p$ and $p, \neg p \nvDash p$.

Why is it objectionable to proceed thus? There may very well be a problem with our semantic metatheory and the problem may cause the semantic consequence relation to be inconsistent. Similarly for the syntactic consequence relation. Such inconsistency should be taken seriously. But for this inconsistency to occur, it is neither sufficient nor necessary that both *p* and $\neg p$ are consequences of the premise set. Incidentally, spreading inconsistency over the levels blocks the advantages of the consistent description from T4.[29]

T7 Where $T = \langle\Gamma, \mathbf{L}\rangle$ is rejected, modified or replaced, the rejection, modification and replacement may affect L as much as Γ.

The main argument for T7 was already adduced for T2: if the true logic exists, it depends in part on the described domain. Actually, a similar argument pertaining to the language rather than the logic was presented ages ago by Hempel (1958).

[29] Somewhat similar, although less objectionable, Graham Priest defines "*A* is false" as "$\neg A$ is true". Note that it is possible to use "*A* is false" as the classical negation of "*A* is true" and $M \nVdash A$ (*M* falsifies *A*) as the classical negation of $M \Vdash A$ (*M* verifies *A*). Even less objectionable, but still related, is that, in **LP** and **CLuNs** and similar logics, $\neg(A \wedge \neg A)$ 'means' $\neg A \vee \neg\neg A$ rather than the denial of the contradiction $A \wedge \neg A$.

The lesson with respect to logical pluralism is small but novel: there is no need for the underlying logic(s) to be stable over time.

6. In conclusion

What preceded should be seen as an attempt to sketch the role of paraconsistent logics within a pluralistic outlook. That I reject, and argued against, classical, dialetheist, intuitionist, or other monistic outlooks should not be misunderstood. Disagreeing does not prevent one from learning from the opponent. Nor should it prevent one from encouraging the opponent to elaborate and correct his or her outlook. As far as I am concerned, this holds especially for dialetheists. What matters is the final full system. A final full system is very remote today, for every outlook. All we have today are scattered knowledge bunches, each with many parts unexplored. So the situation described as the general epistemological situation in Section 2, is exactly the situation in the philosophy of logic: a patchwork of knowledge bunches, with a variety of logics in all three functions. New outlooks often start by taking over most patches from an older outlook, rejecting some other patches of that outlook. The missing bits are either filled out by the result of new ideas and research, or are taken from a different older outlook. Now and then new results from one of the competing outlooks are so basic that all outlooks add the result to their own patchwork, possibly only after a certain period of time and possibly with some reserve.

Paraconsistent logics are a typical example of a result now accepted by all outlooks. They are at least accepted as formal systems, respectively closure operations. Thirty years ago some logicians raised even technical objections to the formal systems and treated scholars interested in paraconsistent systems slightly worse than pedophiles. Meanwhile, the paraconsistent outlooks—there are indeed quite a few—had originated as described above. All outlooks rejected the classical outlook. Yet, some replaced **CL** by a different 'single true logic', or stayed convinced that locating the true logic is a crucial adequacy condition. Others gave up logical monism. While such choices are clearly inspired by our ideological preferences, it would be reckless to consider our own choice as beyond criticism. By the time that our patchwork gets unified, if it ever will, some present theses, and even present patches, will be rejected. Moreover, and as suggested before, we need other outlooks in order to improve and strengthen ours. On the one hand, adherents of other outlooks will be eager to locate the weak spots in ours. On the other hand, whether adherents of a different outlook are able to solve their problems may provide crucial information for modifying our own outlook. For this reason, for example, I am eager to see whether dialetheists are able to remove the weak spots that I consider as critical for the viability of their outlook.

Let us quickly glance at the questions raised by the editors in their call for papers—these may or may not be retained in the introductory paper for the present volume. I do not see any particularly interesting connection between pluralism and inconsistency toleration. Nor do I think that pluralism entails a specific type of commitment towards inconsistency toleration. As was argued in Section 5, unlike the monist, the pluralist has room for a final and stable end state that is fully unified but nevertheless contains classical theories along with paraconsistent ones. However, while the pluralist can allow for more possibilities than the monist, there is no need for the pluralist to adhere to a logically 'mixed' end state.

I definitely do not think that particular inconsistency toleration commitments entail a particular kind of scientific pluralism. In my view, we should try to replace inconsistent knowledge chunks by consistent ones, we should strive towards unification of the knowledge chunks, we should try to maximally integrate them, for example by devising theories from which several present theories may be derived, and the resulting structure will be more preferable as it is simpler (in a sense to be determined). However, the values advertised in the previous statement cannot be realized now. Possibly they will never be realized. The interplay between the complexity of the world and the confines of human information handling capacities may impose severe restrictions; they clearly do now and they may do forever. So inconsistency should be tolerated where that interplay requires it, and similarly for giving in on coherence or unification. We should try to realize each of those values and avoid to connect, for example, a lack of consistency with a lack of coherence.

We obviously need to distinguish between different types of inconsistency toleration commitments. Especially important is the distinction between 'finally' inconsistent theories as compared to transitionally occurring inconsistency. The transitional ones, which regularly crop up in one or other knowledge chunk, are the ones we try to eliminate. Everyone who leaves room for finally inconsistent theories—to avoid misunderstanding, I state that I do—is in need of criteria for the distinction.

The question which inconsistent but non-trivial scientific theories are well understood by which types of paraconsistent approaches cannot be answered in this paper. I have hinted to a few rules of thumb in the last footnote of Section 3, but we definitely do not know enough about the matter at present.

Allow me to end this paper with a comment on what I take to be a justified form of optimism. The advent of paraconsistent logics has opened a wide domain of research, which has branches in mathematics, in the empirical sciences, in scientific methodology–independent of this also in the humanities, including the arts, culture, and philosophy in the broad sense. Some offspring of research in paraconsistency, like adaptive logics, have led to theories that, on the one hand, have a much higher complexity than the usual semi-recursive theories and, on the other hand, handle forms of complexity that are very different from the ones handled by second order logics. Indirectly, this evolution is an argument in favor of logical pluralism. Now that those new types of theories came into reach, next to new types of reasoning, why should we rewrite the old theories, along with old reasoning forms, and forge all into the new formats? Knowledge would be transparent and simple if one logic would perform all functions. Yet, there is no need for a single logic, and certainly no urgency. There are different ways to integrate theories into a coherent unity.

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REFERENCES

Anderson, A.R., Belnap, N. D., Jr., and Dunn, J.M. (1992) *Entailment. The Logic of Relevance and Necessity*, volume 2. Princeton University Press.

Batens, D. (In preparation) Adaptive fregean set theory.

Batens, D. (2016) Spoiled for choice? *Journal of Logic and Computation*, 26(1):65-95, 2016. E-published 2013: doi:10.1093/logcom/ext019.20

Batens, D. (1985) Meaning, acceptance, and dialectics. In Joseph C. Pitt, editor, *Change and Progress in Modern Science*, 333-360. Reidel, Dordrecht.

Batens, D. (1999) Contextual problem solving and adaptive logics in creative processes. *Philosophica*, 64:7-31.

Batens, D. (2002) Towards the unification of inconsistency handling mechanisms. *Logic and Logical Philosophy*, 8:5-31.

Batens, D. (2003) A formal approach to problem solving. In Claudio Delrieux and Javier Legris, editors, *Computer Modeling of Scientific Reasoning*; pages 15-26. Universidad Nacional del Sur, Bahia Blanca, Argentina.

Batens, D. (2004) The need for adaptive logics in epistemology. In Dov Gabbay, Shahid Rahman, John Symons, and Jean Paul Van Bendegem, editors, *Logic, Epistemology and the Unity of Science*, pages 459-485. Kluwer Academic

Batens, D. (2007) Content guidance in formal problem solving processes. In Olga Pombo and Alexander Gerner, editors, *Abduction and the Process of Scientific Discovery*, pages 121-156. Centro de Filosofía das Ciências da Universidade de Lisboa, Lisboa.

Batens, D. (2008) On possibilities and thought experiments. In Robert Almeder, editor, Rescher Studies. *A Collection of Essays on the Philosophical Work of Rescher*, pages 29-57. Ontos Verlag, Frankfurt.

Batens, D. (2012) It might have been Classical Logic. *Logique et Analyse*, 218:241-279.

Batens, D. (2014a) Adaptive logics as a necessary tool for relative rationality. Including a section on logical pluralism. In Erik Weber, Dietlinde Wouters, and Meheus, J, editors, Logic, *Reasoning and Rationality*, pages 1-25. Springer, Dordrecht, 2014.

Batens, D. (2014b) The consistency of Peano Arithmetic. A defeasible perspective. In Patrick Allo and Bart Van Kerkhove, editors, *Modestly Radical or Radically Modest. Festschrift for Jean Paul van Bendegem on the Occasion of His 60th Birthday*, volume 24 (sometimes 22) of Tributes, pages 11-59. College Publications, London.

Batens, D. (2015) Tutorial on inconsistency-adaptive logics. In Jean-Yves Beziau, Mihir Chakraborty, and Soma Dutta, editors, *New Directions in Paraconsistent Logic*, volume 152 of Springer Proceedings in Mathematics & Statistics, pages 3-38. Springer.

Batens, D. Provijn, D. (2003) Pushing the search paths in the proofs. A study in proof heuristics. *Logique et Analyse*, 173-175:113-134.

Batens, D., De Clercq, K., Verdée, P., and Meheus, J. (2009) Yes fellows, most human reasoning is complex. *Synthese*, 166:113-131.

Batens, D., Mortensen, Priest, G. and Van Bendegem, J.P. (edts) (2000). *Frontiers of Paraconsistent Logic.* Research Studies Press, Baldock, UK.

Benferhat, S., Dubois, D., Prade, H. (1997) Some syntactic approaches to the handling of inconsistent knowledge bases: A comparative study. Part 1: The flat case. *Studia Logica*, 58:17-45.

Benferhat, S., Dubois, D., Prade, H. (1999) Some syntactic approaches to the handling of inconsistent knowledge bases: A comparative study. Part 2: The prioritized case. In Ewa Or Łowska, editor, *Logic at Work. Essays Dedicated to the Memory of Helena Rasiowa*, pages 473-511. Physica Verlag (Springer), Heidelberg, New York.

Brady, R. (2006) *Universal Logic.* CSLI Publications, Stanford, Cal., 2006.

Brown, B., Priest, G. (2004) Chunk and permeate, a paraconsistent inference strategy. Part i: The infinitesimal calculus. *Journal of Philosophical Logic*, 33:379-388.

Carnap, R. (1947) *Meaning and Necessity.* University of Chicago Press, Chicago, 1947.

Hempel, C. G. (1958) The theoretician′s dilemma: a study in the logic of theory construction. In H. Feigl, M. Scriven, and G. Maxwell, editors, *Minnesota Studies in the Philosophy of Science*, volume 2. University of Minnesota

Laudan, L. (1977) *Progress and its Problems.* University of California Press, Berkeley, 1977.

Laudan, L. (1990) *Science and Relativism.* University of Chicago Press, Chicago.

Meheus, J. (1995) Nieuwe perspectieven voor het begrijpen en bevorderen van creativiteit. *Mores*, 40:164-178.

Meheus, J. (2000a) Analogical reasoning in creative problem solving processes: Logico-philosophical perspectives. In Fernand Hallyn, editor, *Metaphor and Analogy in the Sciences*, pages 17-34. Kluwer, Dordrecht, 2000.

Meheus, J. (2000b) An extremely rich paraconsistent logic and the adaptive logic based on it. In Batens et al. (2000), pages 189-201.

Meheus, J. (1999a) Deductive and ampliative adaptive logics as tools in the study of creativity. *Foundations of Science*, 4:325-336.

Meheus, J. (1999b) Model-based reasoning in creative processes. In Lorenzo Magnani, Nancy Nersessian, and Paul Thagard, editors, *Model-Based Reasoning in Scientific Discovery*, pages 199-217. Kluwer/Plenum, Dordrecht.

Rescher, N. (1964) *Hypothetical Reasoning*. North-Holland, Amsterdam.

Rescher, N. (2003) *Imagining Irreality.* Open Court, Chicago and La Salle, Illinois.

Rescher, N. (2005) *What If?* Transaction Publishers, New Brunswick, New Jersey.

Rescher, N., Manor, R. (1970) On inference from inconsistent premises. *Theory and Decision*, 1:179-217.

Routley, R. (1982) *Relevant Logics and their Rivals, volume 1.* Ridgeview, Atascadero, Ca.

Schotch, P. K., Jennings, R.E. (1989) On detonating. In Graham Priest, Richard Routley, and Jean Norman, editors, *Paraconsistent Logic. Essays on the Inconsistent*, pages 306-327. Philosophia Verlag, Munchen.

Schutte, k. (1960) *Beweistheorie.* Springer, Berlin.

Shapere, D. (2004) Logic and the philosophical interpretation of science. In Paul Weingartner, editor, *Alternative Logics. Do sciences need them?* pages 41-54. Springer, Berlin, Heidelberg.

Verdée, P. (2009) Adaptive logics using the minimal abnormality strategy are $\prod_1^1 -\mathrm{complex}$. *Synthese*, 167:93-104, 2009.

Verdée, P. (2013a) Non-monotonic set theory as a pragmatic foundation of mathematics. *Foundations of Science*, 18:655-680.

Verdée, P. (2013b) Strong, universal and provably non-trivial set theory by means of adaptive logic. *Logic Journal of the IGPL*, 21(1):108-125.

Verhoeven, L. (2001) All premises are equal, but some are more equal than others. *Logique et Analyse*, 173-174-175:165-188.

Verhoeven, L. (2003) Proof theories for some prioritized consequence relations. *Logique et Analyse*, 183-184:325-344.

Vickers, P. (2013) *Understanding Inconsistent Science.* Oxford University Press, Oxford.

Weber, Z. (2010) Transfinite numbers in paraconsistent set theory. *Review of Symbolic Logic*, 3:71-92.

Weingartner P. (2000) Reasons for filtering Classical Logic. In Batens et al., pages 315-327.