

Diodorus Cronus: Modality, the Master Argument and Formalisation

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ABSTRACT

In his Master Argument, Diodorus used the premisses that "Every past truth is necessary" and "The impossible does not follow from the possible" to conclude "Nothing is possible that neither is true nor will be." His ultimate aim was to defend a definition of the possible as that which either is true or will be. Modern scholars have deployed a wide variety of formal notations in order to formalise the ideas of Diodorus. I show how, with one exception, those notations are simply not adequate for this purpose.

A reviewer of an encyclopedia of philosophy once wrote:

As one who has pretensions to being educated in philosophy, I was distressed to discover that there was a "master argument" due to Diodorus Cronus (who died early in the third century B.C.) of which I knew nothing. Still worse, the argument turned out to be the verbal equivalent of a Rubik's Cube, and I could make nothing of it.¹

I wrote the encyclopedia article that so baffled the reviewer. Here I would like to make some amends. Unfortunately however I will not always be able to be as clear as I would wish, for I will be criticising various attempts to formalise Diodorus by alternatives to the most obvious and straightforward formalism.

1. THE MAIN TEXTS

According to our amplest ancient report, Epictetus 2.19.1:

The Master Argument was apparently based on some such assumptions as these. There is a mutual conflict of these three with each other:

Every past truth is necessary;

The impossible does not follow from the possible; and

Something is possible that neither is true nor will be.

Seeing this conflict, Diodorus relied on the plausibility of the first two to establish:

Nothing is possible that neither is true nor will be.

Diodorus' purpose was to establish a definition of the possible, whereby the possible is that which either is true or will be. And this definition was one of a family of such definitions.

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¹ (Meynell, 1996), review of (Honderich, 1996).



According to our amplest ancient report of these, Boethius, *Commentary on the De Interpretatione of Aristotle* 234.22-26:

Diodorus defines as possible that which either is or will be; the impossible as that which, being false, will not be true; the necessary as that which, being true, will not be false; the non-necessary as that which either already is or will be false.

Our direct evidence about the Master Argument does not extend much further than this. We do indeed learn from elsewhere that the Master Argument was, at least in some circles, a topic of conversation during and after dinner (Plutarch, *Moralia* 133 b-c and 615a, Aulus Gellius 1.2.4, Epictetus 2.19.8). We are advised to infer from this that the Master Argument “cannot have been unduly complex in structure”.² The advice, correct though it is, does little to constrain reconstructions of the Master Argument: for there are dinner tables (*experto credite*) at which people discuss Gödel’s proofs of his Incompleteness Theorems, and Wiles’ proof of Fermat’s Last Theorem. Equally unhelpful in reconstructing the Master Argument is the assertion of Michael Psellus *Theologica* 3.129-135 that the Master Argument got its name as a conceit of a pattern then standard: the Heaper Argument was an argument about heaps that itself heaped up many inferences (“One grain does not make a heap; if one grain does not make a heap, then two grains do not make a heap;...; so ten thousand grains do not make a heap”); the Horned Argument was an argument about horns that itself presented victims with the horns of a dilemma (“Either you have lost your horns or you have not lost your horns;...; either way, you have at some time had a cuckold’s horns”); so too the Master Argument was a masterly argument about mastership. The consequence is that an adequate reconstruction of the Master Argument should be applicable to mastership (“Suppose that it is possible for Dion to be in charge, even though he is not now nor ever will be”), and should not be conspicuously weak. This consequence cannot be denied. But it cannot rule out any reconstruction that would otherwise be plausible.

There are other texts from the ancient world with a bearing on the Master Argument. But their bearing is in each case fairly indirect. We will encounter them in later sections of this article.

2. FOUR FORMALISMS

Scholars who attempt to reconstruct the Master Argument often do so by formalisation. They have very different beliefs about what sort of formalism is appropriate. My own belief is that, to formalise Diodorus’ ideas, the only appropriate formalism is that of Arthur Prior. I will here expound that formalism, and explain why three of its rivals are of no help in formalising Diodorus’ ideas.

² (Long and Sedley, 1987), p. 233.



2.1 TENSE-CUM-MODAL LOGIC IN THE STYLE OF PRIOR

This is the formalism devised by Arthur Prior, and used in his “Analysis of the Master-argument of Diodorus”, in his (Prior, 1967) pp. 32-4. The atomic formulae of this formalism are tensed sentences (e.g. “Dion is ruling”) that may be combined with truth-functional operators, tense operators P and F (“It has been the case in the past that” and “It will be the case in the future that”), and modal operators M and L (“It is possible that” and “It is necessary that”), in any order, to make further tensed sentences (e.g. “It never has been possible that Dion will always be ruling”).

Those who seek a formal semantics for this notation can develop one along these lines: a model contains a set of possible moments of time, one of which is singled out as the actual present moment; on this set are defined two relations, the relation of being later than and the relation of being accessible from; the sentence Fp is true at a possible moment if and only if the sentence p is true at some later possible moment; the sentence Pp is true at a possible moment if and only if the sentence p is true at some earlier possible moment; the sentence Mp is true at a possible moment if and only if the sentence p is true at some possible moment accessible from that moment; the sentence Lp is true at a possible moment if and only if the sentence p is true at every possible moment accessible from that moment; a sentence is true in the model if and only if it is true at the actual present moment of the model.

2.2 TENSE-CUM-MODAL LOGIC IN THE STYLE OF GASKIN

This is the formalism devised by Richard Gaskin in (Gaskin, 1999).³ Gaskin’s tense logic draws on a distinction between sentence-radicals and sentences proper. Sentence-radicals are the lower-case letters p, q, r , etc., and all formulae that can be compounded from sentence-radicals by truth-functional connectives, the past tense operator P , and the future tense operator F . No sentence-radical is a sentence proper, and so no sentence-radical has a truth-value. A sentence-radical may be converted into a sentence proper by prefixing it with an N (the “closing operator”, to be pronounced as “It is *now* the case that”). And sentences proper are all the formulae that can be compounded from sentences proper by truth-functional connectives, and the modal operators L, M and Q for necessity, possibility and contingency. That, at any rate, is the official notation. For practical purposes however, Gaskin usually omits the N s. This is because, when we can add N s to a string of symbols to produce a sentence of the official notation, the various sentences that we produce are all equivalent.

Gaskin gives no semantics for this notation. Nor is it at all easy to see how a semantics might be developed.

³ Gaskin’s tense-logical version of the Master Argument was first presented in his (Gaskin, 1996), which was a response to my (Denyer, 1996), pp. 166-180, which was a review of his (Gaskin, 1995), which on pp. 290-1 reconstructed the Master Argument in a formalism based on predicate calculus.



The closing operator N makes Gaskin's favoured formalism very different from any standard modal-cum-tense logic. For a standard modal-cum-tense logic would count as well-formed various formulae in which a modal operator occurs within the scope of a tense operator: FMp , PLp and the like. There is however no way of inserting N s into such formulae so as to make them well-formed formulae of Gaskin's official notation, for "the closing operator must be placed *inside* the scope of modal operators but *outside* the scope of tense operators" ((Gaskin, 1999), 211). This means that Gaskin's favoured formalism cannot express the modal notions employed in, for example, Diodorus' claim that, as e.g. Sextus Empiricus *Pyrrhoniae hypotyposes* 2.110 puts it, a conditional is sound if and only if: "it neither was able nor is able to have a true antecedent and false consequent".

It is unlikely that Diodorus, although using these modal notions in his account of conditionals, should then use different modal notions in his Master Argument. It is therefore unlikely that Gaskin's favoured formalism can express the modal notions used in the Master Argument.

2.3 QUANTIFIED MODAL LOGIC WITH INDEXICALS

This is the formalism favoured by, for example, (Vuillemin, 1996).⁴ Its basis is the first-order predicate calculus, with moments of time taken as the domain of quantification. It includes not only names of constant denotation (e.g. "noon-GMT-on-22.8.2005"), but also names— or quasi-names—whose denotation can vary (e.g. "now", "this time tomorrow"), and combines these with predicates of times to make sentences that are liable to vary between truth and falsehood. Thus this formalism would render the present tense "Dion is ruling" by a formula to be read as "Now is-a-moment-during-rule-by-Dion", and it would render the past tense "Dion has been ruling" by a formula to be read as "For some x , x is-a-moment-during-rule-by-Dion, and now is after x ." To this basis the formalism adds modal operators that produce formulae when applied to a pair of expressions, of which one is a name for a time, and the other a formula. An example might be "It is at the present moment necessary that noon on 1 January 1999 is-a-moment-during-rule-by-Dion."

This formalism can give no apt rendering of the first assumption of the Master Argument that "Every past truth is necessary." For it can render the first assumption only along such lines as, most simply:

If x is before now and Fx , then it is at the present moment necessary that Fx ,

or a generalisation of this, such as:

If x is before y and Fx , then it is at y necessary that Fx ,

⁴ I have reviewed this at greater length in (Denyer, 1998).



or, more elaborately still:

If for some x , x is before now and Fx , then it is at the present moment necessary that for some x , x is before now and Fx .

But, on any such rendering, the first assumption has the grotesque implication that all truths are necessary. Let us see how this works in detail for the simplest such rendering, and leave as an exercise for the reader the extension to the other renderings. “55 B.C. = 55 B.C.” is logically true. Hence any formula is logically equivalent to its conjunction with “55 B.C. = 55 B.C.” But 55 B.C. is before now. So any formula is logically equivalent to a proposition mentioning some time before now. In particular therefore, any true formula will be logically equivalent to some formula that is now necessary. But a formula has the same modality as any proposition to which it is logically equivalent. So any true formula will be necessarily true.

No less grotesque is the way that this formalism would render Diodorus’ definitions of modal concepts. For this formalism makes those definitions imply both that every truth is necessary, and that none is. To see this, let us recall that any formula p is logically equivalent to, and has the same modality as, the formula $p \ \& \ now = now$. From this latter formula, we can produce the open sentence $p \ \& \ x = x$, which is true of all times if p itself is true. So p implies a formula that may be read as “ p and now is identical to itself, and for every time later than now, p and that time is identical to itself.” Such a formula will be a rendering in this formalism of “It is and always will be true that $p \ \& \ now = now$.” But Diodorus defined the necessary what is and always will be true. So, if we are to accept this formalism, p implies that p is necessary, and every truth is a necessary truth. Moreover, from $p \ \& \ now = now$, we can also produce the open sentence $p \ \& \ now = x$. So Diodorus’ definition of necessity will equate “It is now necessary that p ” with a formula to be read as “ p and now is identical to now, and for every time later than now, p and that time is identical to now.” But no such formula is true, and hence no truth is a necessary truth.

2.4 QUANTIFIED MODAL LOGIC WITHOUT INDEXICALS

This is the formalism favoured by Nicholas Rescher in (Rescher, 1966). This is like the formalism favoured by Vuillemin, except that it allows as names for times only those that, like “noon-GMT-on-22.8.2005”, are of constant denotation. Because it is based on predicate calculus, it has all the faults of Vuillemin’s formalism. And because not one of its formulae is capable of varying between truth and falsehood, it has a distinctive fault of its own. For changes of truth-value are envisaged in the Master Argument itself (e.g. “Nothing is possible that neither is true nor will be”), in Diodorus’ own definitions of modal concepts (e.g. “the non-necessary as that which either already is or will be false”), and in his own teaching that, as Sextus Empiricus *Adversus mathematicos* 10.97-99 puts it:

it is possible to have true pasts whose presents are false. E.g. suppose someone married one year earlier, and someone else one year later. So in their case the proposition “These men married” is, being past, true; whereas “These men are marrying” which is a present is false. For when the one



was marrying, the other was not yet marrying. And “These men are marrying” would have been true of them if they married simultaneously. So it possible for a true past to have a false present. Also like this is “Helen had three husbands”. For neither when she had Menelaus as her husband in Sparta, nor when she had Paris in Ilium, nor when, on his death, she married Deiphobus, is the present “She has three husbands” true, although the past “She had three husbands” is true.

Diodorus was in no way eccentric to envisage such changes of truth-value. Carneades took them for granted, when he gave the oversimplified account of tensed statements that is reported in Cicero *De fato* 27 as:

Just as we call true those past-tense propositions whose present was true at some previous time, so we should call true those future-tensed propositions whose present will be true at some later time.

Such changes are taken for granted also by Chrysippus, as reported in Cicero *De fato* 14:

For all truths in past tenses are necessary, as Chrysippus declares, in disagreement with his master Cleanthes, since they are immutable, and being past-tensed cannot change from true to false.

For although Chrysippus was happy to reason in this way that all past truths are necessary, he nevertheless maintained that some truths are contingent. Other examples could be given.⁵

When an entire philosophical culture is so ready to believe that truth-values can change, it is hard to accept that we are being faithful to their ideas when we formalise them in a formalism that expressly precludes such changes.

3. THE FIRST ASSUMPTION OF THE MASTER ARGUMENT

The first assumption of the Master Argument is reported by Epictetus as “Every past truth is necessary.” Three interpretations of this assumption deserve mention here, of which only the first is plausible.

3.1 PRIOR ON PAST TRUTHS

Prior’s interpretation depends on the thought that we should not count as past truths absolutely all truths that somehow or other involve the past tense. For example, “Claire has never yet had a son” involves the past tense, but it is quite unlike anything that Carneades had in mind when he said that those past statements are true whose presents have been true, and it is quite unlike anything that Chrysippus had in mind when he said that all past truths are

⁵ Perhaps the most striking would be Aristotle *Categories* 4a16-4b5 and Alexander of Aphrodisias *De fato* 177.15-22, passages where the author agrees without argument that propositions can change truth-value, although it would be in many respects more convenient for him if no such change were possible.



necessary, since they cannot change from truth to falsehood; for the fact that “Claire has no son” has been true hardly means that she has never had a son, and the fact that she has never yet had a son hardly means that she never will. Indeed, it is evident that when Carneades and Chrysippus talked of past truths they had in mind only truths of the form Pp . If we take this as our guide, we should formalise the first assumption as $Pp \rightarrow LPP$.

Could this be all that the first assumption means? Diodorus, we have seen, offered “These men have married” and “Helen had three husbands” as past truths that manifestly are not of the form Pp , for they are true even though “These men are marrying” and “Helen has three husbands” never have been true. Yet these past truths look just as necessary as past truths of the form Pp , like “This man has married” and “Menelaus was Helen’s husband.” Should we modify our formalisation of the first assumption to allow for this? Probably not. For if a proposition of the form “These men have married” is true, then there will be truths of which it is a logical consequence, and which are necessary by the principle that $Pp \rightarrow LPP$; for it will be a logical consequence of some truths of the form “This man has married.” But a logical consequence of necessary truths is itself necessary. So any truth of the form “These men have married” will be necessary, according to the first assumption as we have formalised it, quite without any modification. The same holds also for “Helen had three husbands.” So we have no reason here to take the first assumption to be other than $Pp \rightarrow LPP$.

3.2 WEIDEMANN ON PAST TRUTHS

Hermann Weidemann takes the first assumption to claim more than that $Pp \rightarrow LPP$.⁶ He takes it to be, in effect, the claim that $p \rightarrow L(p \vee Pp)$. For he takes past truths to be, not truths of the form Pp , but truths of the form $p \vee Pp$; so he takes the first assumption to be the claim that $(p \vee Pp) \rightarrow L(p \vee Pp)$, which is equivalent to the conjunction of $p \rightarrow L(p \vee Pp)$ with $Pp \rightarrow L(p \vee Pp)$, which is an immediate consequence of $Pp \rightarrow LPP$, which is equivalent to $Pp \rightarrow L(Pp \vee Pp)$, which results from substituting Pp for p in $p \rightarrow L(p \vee Pp)$. It would be convenient if the first assumption does claim that $p \rightarrow L(p \vee Pp)$, for this claim seems no less plausible than $Pp \rightarrow LPP$, and with this claim as its first assumption the Master Argument would be incontestably valid: suppose some proposition is possible that neither is nor ever will be true; then from the actual present moment (call it a) there is accessible some moment (call it m) that is neither identical to nor later than a ; now let p be a proposition true at a , but at no other moment; it follows that $\neg(p \vee Pp)$ is true at m , and therefore that $L(p \vee Pp)$ is false at a ; and this contradicts the claim that $p \rightarrow L(p \vee Pp)$. But although convenient, Weidemann’s rendering of the first assumption looks implausible. For there is no sign that any ancient classified as past truths, not truths of the form Pp , but those of the form $p \vee Pp$.

⁶ See (Weidemann, 2008), pp. 131-148, at p. 141. The point should not be obscured by the fact that Weidemann would formalise the first assumption by a formula that looks just like our $Pp \rightarrow LPP$. For Pp in his notation is the same as $p \vee Pp$ in ours.



3.3 GASKIN ON PAST TRUTHS

Gaskin too takes the first assumption to claim more than that $Pp \rightarrow LPp$. For he takes past truths to include, not only truths of the form Pp , but also truths of the form $\neg Pp$, and so takes the first assumption to claim also that $\neg Pp \rightarrow L\neg Pp$. Gaskin therefore makes Diodorus' notion of past propositions very different from the one we saw Chrysippus and Carneades take as obvious. However, he offers no evidence that anyone apart from Diodorus had such a notion of past propositions. Nor does he say why anyone at all, whether Diodorus or another, should find plausible an assumption which claims that $\neg Pp \rightarrow L\neg Pp$, when truths of the form $\neg Pp$ can so readily change from truth to falsehood.

There is, on Gaskin's account, more to the first assumption even than this. For $Pp \rightarrow LPp$ and $\neg Pp \rightarrow L\neg Pp$ together amount to $\neg QPp$, which implies $Q(Pp \vee p \vee Fp) \rightarrow Q(p \vee Fp)$, which is only one half of the biconditional $Q(Pp \vee p \vee Fp) \leftrightarrow Q(p \vee Fp)$ whereby Gaskin formalises the first assumption. Thus, on Gaskin's account, the first assumption claims additionally that $Q(p \vee Fp) \rightarrow Q(Pp \vee p \vee Fp)$.

Gaskin has vacillated about this additional claim. At one stage, he said it was "uncontroversial" ((Gaskin, 1996), 190); at a more recent stage, he said it was "not guaranteed to be true" ((Gaskin, 1999) p. 216). His second thoughts were wiser. For in making this additional claim, the first assumption rules out cases like these: Dion has been in power (where p is "Dion is in power", this means that Pp , and therefore that LPp , and therefore that $L(Pp \vee p \vee Fp)$, and therefore that $\neg Q(Pp \vee p \vee Fp)$), but was deposed yesterday; he might yet be back in power (this means that MFp and therefore that $M(p \vee Fp)$), but he is not in power at the moment, and might never be in power again (this means that $M\neg(p \vee Fp)$, and therefore that $Q(p \vee Fp)$). Neither at the earlier nor at the later stage has Gaskin offered any explanation of why, if the first assumption makes such a claim, Diodorus should have been able to rely on its plausibility. Nor has he offered any explanation of how to derive such a claim from Epictetus' wording about the necessity of all past truths.

4. THE SECOND ASSUMPTION OF THE MASTER ARGUMENT

The second assumption of the Master Argument is reported by Epictetus as "The impossible does not follow from the possible." The obvious interpretation is that Diodorus was reasoning from the assumption that if p is possible and q is impossible, then q does not follow from p .

Gaskin however formalises the second assumption in a way quite different from this. For he treats it as formulating and endorsing this rule of inference:

If $(A \ \& \ \neg A)$ follows from A taken together with some other assumptions, then $\neg QA$ follows from those other assumptions alone.



Here is an example of how to apply this rule of inference: since $p \ \& \ \neg p$ follows from p taken together with $\neg p$, then $\neg Qp$ follows from $\neg p$ alone. Alternative formulations of essentially the same rule of inference would be that Lp follows from p , and that p follows from Mp .

Gaskin does not attempt to explain how to extract his rule of inference from the wording in Epictetus.

There is however one difficulty about Gaskin's interpretation of the second assumption to which he is alert. It is that Gaskin's rule of inference seems to imply that every truth is a necessary truth, which would make the rule lack the plausibility of which Epictetus speaks. Gaskin's solution to this difficulty is that the full panoply of classical logic was not widely accepted in Diodorus' day, and that what follows from his rule of inference by principles that were widely accepted is not the objectionably fatalistic claim that the conditionals $Mp \rightarrow p$ and $p \rightarrow Lp$ are always true. Specifically, the rule of inference is tantamount to the principles that Lp follows from p and that p follows from Mp ; Aristotle, who was no fatalist, accepted these principles; to get from these principles to the objectionably fatalistic claims, we need the principle of Conditional Proof; but the principle of Conditional Proof was not universally accepted; it would have been contested by, among others, Aristotle ((Gaskin, 1996), 186-9; (Gaskin, 1999), 215).

Gaskin's solution to this difficulty faces a further difficulty of its own. It is that, whatever Aristotle may have thought, Diodorus himself and his successors had no hesitation about accepting the principle of Conditional Proof. It was the consensus among them all, we learn from Sextus Empiricus *Against the Learned* 8.112, that: "a conditional is sound whenever its consequent follows from its antecedent". Their only dispute was over what it is for one proposition to follow from another. Indeed, Gaskin himself points out (his ((Gaskin, 1996), 191; (Gaskin, 1999), 216 n. 29) that, in his own reconstruction, Diodorus uses Conditional Proof and kindred principles.

5. THE THIRD ASSUMPTION OF THE MASTER ARGUMENT

The third assumption of the Master Argument was given by Epictetus "Something is possible that neither is true nor will be," where "*dunaton*" is the Greek word that I translate as "possible". The obvious way to formalise this in standard modal-cum-tense logic is as saying that for some p , $Mp \ \& \ \neg(p \vee Fp)$.

Gaskin formalises the third assumption in his notation as saying that for some p , $Q(Pp \vee p \vee Fp) \ \& \ \neg(p \vee Fp)$. It says, in other words, that for some p , it is possible that p be true sometime, it is possible that p be true never, and p neither is nor will be true. In effect then, Gaskin formalises the statement that p is *dunaton* by the formula $Q(Pp \vee p \vee Fp)$. How reasonable is this? As evidence that "*dunaton*" can bear such a meaning, Gaskin cites the way that Aristotle uses it and its cognate "*dunasthai*" in his discussion of two-way capacities in *De Interpretatione* 12-13 ((Gaskin, 1999), 213). A typical passage would be 21b12-15, where Aristotle says:

The same thing appears to have a capacity both for being and for not being. For what is capable of being cut or of walking is also capable of not being cut or of not walking. The reason is that whatever is in this fashion capable is not always actually operating, so that the negation too will be present in it.



However, such passages do not support Gaskin's interpretation of "*dunaton*". For Aristotle's idea is that if a thing has the two-way capacity of walking, then it is possible that the thing walks sometimes, and possible also that the thing sometimes fails to walk. In consequence, the existence of an Aristotelian two-way capacity for p should be formalized in Gaskin's notation as $M(Pp \vee p \vee Fp) \& M(P\neg p \vee \neg p \vee F\neg p)$; and this is quite different from Gaskin's $Q(Pp \vee p \vee Fp)$.

6. ARE THE THREE ASSUMPTIONS CONSISTENT?

If the three assumptions were as we have interpreted them, then all three assumptions can be true together, and all are true together so long as these conditions are met: every moment earlier than the actual present moment is earlier than every moment accessible from the actual present moment; and some moment accessible from the actual present moment is neither identical to nor later than the actual present moment. Nevertheless, the first and second assumptions come close to ruling out the third, for the first and second assumptions imply that no proposition can be for more than an instant as the third assumption takes some proposition to be: both possible and such that it neither is nor ever will be true. In consequence, we can easily move from accepting the first two assumptions to rejecting the third once we accept the principle that nothing is ever so for only an instant. This principle was accepted by all parties to the debate over the Master Argument.⁷ We can thus explain why, even though the three assumptions of the Master Argument are in fact consistent, those who wanted to accept the third assumption felt constrained by the Master Argument to reject either the first or the second (Epictetus 2.19.2-4).

7. DIODORUS' DEFINITIONS OF THE MODAL TERMS

Upon rejecting the third assumption of the Master Argument, Diodorus concluded that nothing is possible that neither is nor will be true. When combined with the scarcely contestable idea that what is or will be true can be true, this conclusion promptly gives Diodorus' definition of the possible as what is or will be true. And from Diodorus' definition of the possible, it is easy enough to derive what look like his other definitions. For example, since a thing is necessary if and only if its negation is not possible, a thing will be necessary if and only if its negation is not such that it either is or will be true; in other words, a thing will be necessary if and only if it is and always will be true.

⁷ For details, see (Denyer, 1999).



7.1 DIODORUS' DEFINITIONS AND PRIOR'S FORMALISM

All this is very straightforward when formalised in Prior's formalism. The definition of the possible can be formalised as equating Mp with $p \vee Fp$. This means equating $\neg M\neg p$ with $\neg(\neg p \vee F\neg p)$. But $\neg M\neg p$ is equivalent to Lp , and $\neg(\neg p \vee F\neg p)$ is equivalent to $p \& \neg F\neg p$. So the definition of the possible is, by implication, equating Lp with $p \& \neg F\neg p$. The definition of the necessary can be formalised as making precisely that equation. And all is simple.

7.2 DIODORUS' DEFINITIONS AND GASKIN'S FORMALISM

Things are much more complicated on Gaskin's interpretation. According to Gaskin (Gaskin, 1999, 210-13), the sort of possibility of p that Diodorus defined by $p \vee Fp$ cannot be expressed in his notation by Mp . For that formula, recall, is an abbreviation of MNp , and says that it is possible that it is now the case that p , not that it is possible that p . Hence, Gaskin tells us, the possibility of p that Diodorus defined by $p \vee Fp$ is expressed instead by $M(p \vee Fp)$. The sort of necessity that goes with this sort of possibility—the sort of necessity that a proposition has if and only if its negation lacks this sort of possibility—should then be expressed in Gaskin's notation by $\neg M(\neg p \vee F\neg p)$ or some equivalent formula such as $L(p \& \neg F\neg p)$. And if Diodorus had this sort of necessity in mind when he defined the necessary as that which is and always will be true, then his definition of the necessary would be a straightforward consequence of his definition of the possible. However, Gaskin tells us, Diodorus had in mind another sort of necessity altogether: the sort of necessity that he equated with $p \& \neg F\neg p$ is to be expressed in his notation by $L(p \vee Fp)$. So, if, as it is easy to suppose, Diodorus did infer his definition of necessity from the conclusion of the Master Argument, then he was guilty of a fallacy.

It is difficult to assess this argument of Gaskin's. The chief difficulty is in assessing his claims about the proper way to formalise, in his notation, the sorts of necessity and possibility that Diodorus attempted to define. Gaskin does not tell us enough about his notation for us to be able to assess them ourselves. We simply have to take his word for them.

We should however note that if Diodorus' definition of necessity was as Gaskin supposes, then he was an even worse logician than Gaskin ever suggests. For $LF\neg p$ implies $L(\neg p \vee F\neg p)$, which, by the definition of necessity, implies $\neg p \& \neg Fp$, which implies $\neg p$. So, by contraposition, p implies $\neg LF\neg p$, which implies $M\neg F\neg p$, which implies $M(\neg F\neg p \vee F\neg F\neg p)$, which, by the definition of possibility, implies $\neg F\neg p \vee F\neg F\neg p$. So, since I am now alive, it follows that either I will live for ever hereafter, or at least that a time will come when I will live for ever thereafter. Indeed, there follows an even more optimistic conclusion. For suppose that, before I enter into immortality, there will come a time at which I am not alive. Then it would follow, by exactly the same pattern of argument, that either I will never be alive from that time onwards, or at any rate there will come a still later time after which I will never be alive. But neither of these is consistent with our earlier conclusion, that if I have not already entered into immortality, then I will at some time do so. So we were wrong to suppose that, before I enter into immortality, there will come a time at which I am not alive. So I am immortal already—given merely that I am now alive.



7.3 DIODORUS' DEFINITIONS AND THE MEGARICS

Diodorus was sometimes classified as a Megaric.⁸ According to Aristotle *Metaphysics* 1046b29-32, the Megarics held that:

a thing is able to act only when it is acting, and that when a thing is not acting it is unable; e.g. that someone who is not building is unable to build, but someone who is building is able, when he is building, and likewise also for other cases.

Diodorus' view is, in large and obvious ways, different from that of the Megarics.⁹ Yet it is possible to see Diodorus' view as what results from the Megaric view after a few rounds of debate.

The Megarics' initial position, equating the possible with the actual, is refuted by the obvious objection that it rules out all change, for if things never can be different from the way they are, they never will be (Aristotle *Metaphysics* 1047a10-17). In the face of this objection, Megarics can abandon the letter of their initial position while still retaining much of its spirit. Let us imagine them speaking as follows: "There is no way of differentiating falsehoods into those that can be true and those that can't. All falsehoods are alike. They're all impossible." Aristotle then points out that if all falsehoods are impossible, then nothing ever changes. The Megarics can respond: "Very well. Things do change, and so not all falsehoods are impossible. Nevertheless, there is still no way of differentiating falsehoods into those that can be true and those that can't. All falsehoods are still alike. For they're all possible, and the only difference between them is that some will continue to be false for ever, while others will change to be true."

It is just such a response that Aristotle considers in the next round of the debate at *Metaphysics* 1047b3-9:

If the aforesaid [i.e. having no impossible consequences: see *Metaphysics* 1047a24-28] either is or follows from being possible, then it plainly cannot be true to say "The thing is possible; but it never will be"—the upshot of which is that we thus avoid admitting that things are impossible." I mean e.g. if someone—the man who does not reckon that anything is impossible—were to say "It is possible to measure the diagonal; it is just that it never will be measured; because there is nothing to stop a thing that is capable of being or happening from not being either now or in the future."

"Measuring a diagonal" means finding two integers, m and n , such that the diagonal of a square is exactly m/n times as long as the side of the square. A contradiction follows if we suppose that someone has found two such integers: the same number will be both odd and even (see e.g. *Prior Analytics* 41a25-27). After being reminded that some things imply

⁸ For evidence of this fact, and its implications, see (Denyer, 2002).

⁹ The view of the Megarics has been examined in (Makin, 1996).



contradictions, only the utterly incorrigible would continue to maintain, in so many words, that anything can happen, including those things that imply contradictions, and it is just that some things never will. But the corrigible can still maintain, if not exactly this, then at least something very like it.

Think, for example, of the relation between these two philosophies of mind: the Disappearance Theory, whose slogan might be “There are no minds; there are only brains”; and the Identity Theory, whose slogan might be “There are minds; for there are brains, and minds are identical to brains.” In one respect, these two philosophies of mind could hardly be more different: one affirms something that the other denies, the existence of minds. In another respect, these two philosophies of mind amount to variations on a single theme: they both agree that there are no minds apart from brains. Hence someone who starts from the Disappearance Theory, and who then feels constrained to agree that there are minds after all, will naturally move towards the Identity Theory, as the nearest tenable position.

We can imagine a similar development among intellectual descendants of the Megarics whom Aristotle criticized. The development will allow them to maintain all along that the impossible is nothing other than what is not and never will be true, while taking them from the thought that the impossible is nothing whatsoever, to the thought that the impossible is as Diodorus defined it.¹⁰

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¹⁰ An earlier version of this paper was presented at the Symposium Megarense, in August 2005. Thanks are due to the participants in the Symposium, and in particular to Hermann Weidemann, who saved me from several errors.



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